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# The Effects of Metacognitive Writing on Student Achievement in Advanced Placement Calculus


Lindsay M. O'Neal  
*Seattle Pacific University*

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The Effects of Metacognitive Writing on Student Achievement  
in Advanced Placement® Calculus

Lindsay M. O'Neal  
Seattle Pacific University

The Effects of Metacognitive Writing on Student Achievement

in Advanced Placement® Calculus

by

Lindsay M. O’Neal

A dissertation submitted in partial fulfillment

of the requirements for the degree of

Doctor of Education

Seattle Pacific University

2015

Approved by

\_\_\_\_\_  
(Dr. Andrew Lumpe, Ph.D., Chairman, Dissertation Committee)

\_\_\_\_\_  
(Dr. Nyaradzo Mvududu, Ed.D., Committee Member)

\_\_\_\_\_  
(Dr. Robin Henrikson, Ph.D., Committee Member)

Program Authorized to Offer Degree \_\_\_\_\_ School of Education \_\_\_\_\_

Date \_\_\_\_\_

\_\_\_\_\_  
(Dr. Rick Eigenbrood, Ph.D., Dean, School of Education)

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Seattle Pacific University

Abstract

The Effects of Metacognitive Writing on Student Achievement

in Advanced Placement® Calculus

by

Lindsay M. O’Neal

Chairperson of the Dissertation Committee: Dr. Andrew Lumpe, School of Education

Grounded in metacognitive theory (Flavell, 1976) and historical foundations that reach back as far as the writings of Plato (1973), the last few decades have seen an increase in research regarding the impact of metacognitive practice on student learning, often through the use of reflective writing. Studies have focused on a range of aspects, from how to measure metacognition to the effect metacognitive practice has on the academic achievement of students in a variety of subject areas. Specifically with regard to mathematics, researchers have studied the impact of reflective strategies on primary, secondary, and university level students.

The purpose of this study was to explore the impact of reflective writing practice on the achievement of Advanced Placement (AP®) Calculus students in a comprehensive high school setting. This quasi-experimental study utilized a pretest-posttest control group design, with nonrandom assignment of students to the control and experimental groups. The independent variable was the use of reflective writing prompts, completed

only by the experimental group. The non-calculator multiple choice portion of released AP Calculus AB examinations served as the dependent variable.

Descriptive statistics were evaluated to determine if the data met the requirements for parametric analyses. Analysis of covariance was completed to analyze the data for statistically significant differences between the groups. In addition, theme extraction was carried out using Semantria® text analysis software to examine common themes within the reflective student writings as well as Sentiment values for those themes. Finally, Pearson's  $r$  correlation coefficient was calculated to determine any correlation between number of extracted themes and posttest score.

The ANCOVA revealed a statistically significant difference between the groups, but with the control group maintaining a higher mean than that of the experimental group. Common themes in the reflective writing included a variety of calculus concepts addressed during the timeframe of the study. A statistically significant correlation was found between the number of extracted themes and student's score on the posttest.

## Chapter One

### Introduction

#### Overview

“Cogito ergo sum” (*I think therefore I am*) (Descartes, 2004, p. 18). These renowned words, by René Descartes, have echoed across the centuries as philosophical support for human existence and thought. Awareness of our conscious thought is perhaps one of the most distinguishing characteristics of our humanity. Since long before the time of Descartes, human beings tried to understand how the world works. One component of this exploration is the desire to understand oneself and one’s own thoughts. The question then becomes how does one comprehend one’s own understanding, and what cognitive tools can be used to enhance such a process of comprehension?

C.S. Lewis (1944/2001) closed his work, *The Abolition of Man*, with a discussion about what it means to see through. “The whole point of seeing through something is to see something through it...If you see through everything, then everything is transparent. But a wholly transparent world is an invisible world” (p. 81). Being able to look through a window to see the view outside makes absolute sense in most cases. The function of a mirror, however, is in its ability to not be “seen through,” but to reflect the image of the one looking at it. Following Lewis’ argument, if a mirror were to be wholly transparent, we would be invisible to ourselves. In contrast, reflection allows us to better see our own likeness, or at least some remnant of the image of oneself.

Our own likeness, our self, is one component of the concept of knowledge according to Dewey and Bentley (1960), who stated that knowledge depends on the relationship between the person knowing and the object which is to be known. In other

words, knowing something means having a “living relationship” with that object or concept (Palmer, 1983/1993, p. xv). Ultimately, as Jerome Bruner (1996) explained, the true accomplishment of teaching and learning is the organization of ideas in such a way that one knows more than he or she normally should. In this way, knowledge through learning and teaching is intricately linked to the self. How we know ourselves is often through the process of reflection. Bruner (1996), in fact, defined reflection as moving past simple, foundational learning to taking what one has learned and making sense of it.

Palmer (1983/1993) suggested that since our knowledge has a level of ownership over us, then it is important for us know understand that knowledge more deeply. If understanding of our knowledge is truly this important, then it should be one of the key components of educational practice. Wiggins and McTighe (1998) called understanding the ultimate goal of teaching and went on to claim that reflection and persistence are required to grasp often obscure and counterintuitive big ideas. In the calculus classroom, for instance, students are faced with complex mathematical analyses that are often broken down into smaller components, causing many learners of mathematics to see the pieces only in isolation, not allowing them to consider the whole (Tall, 1991). Perhaps reflection could lead to a view of the whole picture for such students. The goal in this study is to explore the theoretical underpinnings of metacognition and reflective writing, to consider research addressing the use of such strategies in a mathematical classroom setting, and to explore the ways in which metacognitive practice and learner-centered reflective writing can support learning.

## **Problem Statement**

Metacognition, as described by Piaget (1950), Dewey (2010), and Brown (1994), and named by Flavell (1979), is a theory that can inform our attempts to find alternative approaches and tools for mathematical learning. Over time, this theory led to a focus on the influence of reflective practice as a component of the application of metacognition in educational settings. Research in metacognition and reflection in mathematics education (Carr, Alexander, & Folds-Bennett, 1994; Desoete, 2007; Lester, Garofalo, & Kroll, 1989; Maqsd, 2007; Naglieri & Johnson, 2000; Wilson, 1986) opened the door for more studies into the effects of using such metacognitive practices to support mathematical learning and the development of problem solving skills. With the increased popularity of college preparatory programs such as Advanced Placement, International Baccalaureate, and Cambridge International Examinations, more students are presented with the opportunity to be exposed to rigorous mathematics courses in high school than in the past. There is a need for additional research on the impact of metacognitive practice on problem solving ability within the realm of high school advanced mathematics courses.

## **Research Questions**

The following research questions are then posed:

1. Does metacognitive writing increase the level of mathematical understanding for Advanced Placement (AP®) Calculus students as shown through measurements of achievement? The null hypothesis is that metacognitive writing has no impact on the mathematical understanding of AP Calculus students.
2. What themes emerge from the reflective journal entries of the AP Calculus students?

3. What is the correlation between the students' number of extracted themes from the reflective journal entries, the sentiment value of those themes, and their scores on the posttest? The null hypothesis is that there is no positive correlation between the number of extracted themes, the sentiment value of the themes, and a students' score on the posttest.



## **Chapter Two**

### **Review of Literature**

#### **Theory**

Thinking about thinking perhaps goes back as early as ancient Greece, if not earlier. Examples of metacognitive strategy use were present at the time of Cicero, when poet Simonides used visualization techniques to recall the seating of attendees at a banquet (as cited in Dunlosky & Metcalfe, 2009). In the writings of Plato (1973), Socrates described the process of thinking to Theaetetus as a conversation the mind has with itself regarding a specific topic of consideration. Much of the more modern approaches to metacognition are likely grounded in the concept of introspection, an idea advocated by Wilhelm Wundt at the turn of the 20<sup>th</sup> Century that involved the observation of one's own actions (Dunlosky & Metcalfe, 2009). William James (1981) also addressed introspection and the exploration of "visual memory" and mental pictures.

In the school setting, students consider multiple academic subjects each day, in addition to the hidden curriculum. One could assume the result is an ongoing discourse within the mind regarding those subjects the student chooses to consider. In a setting such as this one, it might be prudent to heed the warning of Confucius (2007), "Learning without thought is pointless. Thought without learning is dangerous" (p. 21). One would hope that the school environment be one of constant thinking and learning, from the very earliest of childhood experiences through continuing education. Yet the words "critical thinking" are thrown around in some educational settings with no real definition or support of how thinking in such a way might be taught.

According to Piaget (1950), it is important to consider thinking as an active process. Thinking is not necessarily a passive process, but can be an active one as students participate in their own learning. Children, considering their surroundings and experiences, go through a process of construction of objects, space, and time (Piaget, 1954). Their thinking is most certainly active as they conceptually build their realities. Though children may not be fully aware of their cognitive process, these young individuals are thinking and learning from reflection on experiences, constructing their understanding of the world around them. The process of construction is not passive, but inherently active.

Dewey (1938/1997a) also built on this idea of constructing knowledge through experiences, emphasizing the claim that authentic learning really occurs through experience. He noted that reflection on such experiences does not stop at merely a range of ideas, but results in an outcome (Dewey, 2010). Reflective thought expands and builds upon previous thoughts. Each phase of that thought process guides the learner from one step to another (Dewey, 2010), leading to the development of understanding. Like Piaget (1950), Dewey (2010) defined reflective thought as “active” and “persistent” (p. 8). He proposed that there are certain sub-processes involved when it comes to reflective thought, including confusion, doubt, and an investigative stance to shed light on the particular subject being considered. Reflective thinking, then, is active thinking. Dewey (1910/1997b) asserted that to reflect on a suggestion means to mentally hunt for evidence to support an individual’s ultimate conclusions or to determine the suggestion to be invalid. Hunting is certainly an active process, even if the hunt is a mental one.

Built on the historical foundations of thought, introspection, and reflection, Flavell (1976) coined the term “metacognition.” Flavell (1976) defined metacognition as “one’s knowledge concerning one’s own cognitive processes and products or anything related to them” (p. 232) or “knowledge and cognition about cognitive phenomena” (Flavell, 1979, p. 906). Metacognitive action can be broken down into four main components, according to Flavell (1979). These include metacognitive knowledge, metacognitive experience, goals or tasks, and actions or strategies. Brown, Bransford, Ferrara, and Campione (1982) interpreted metacognition from two different perspectives, that of knowledge about one’s cognition and control over those thoughts. The authors defined the knowledge portion of metacognition as late-developing and “stable,” in that cognitive processes can be reflected on and discussed with others (Brown et al., 1982, p. 87). The other side of their metacognitive coin is the part that involves planning, monitoring, and evaluating outcomes in ways that are not always stable (Brown et al., 1982, p. 87). These two ways of viewing metacognition are closely intertwined and not always distinguishable. In later work, Brown (1994) proposed that the fact individuals have knowledge, feelings, and control about their learning is one of the most intriguing aspects of human cognitive processes. She recognized that learners who are effective tend to have insight into their own abilities, strengths, and weaknesses. These learners are also able to access learning strategies based on their own knowledge and experiences in an active manner.

Kluwe (1982) also separated metacognition into two components, that of declarative knowledge and procedural knowledge. One might be able to group Flavell’s (1979) metacognitive knowledge and experience with the knowledge of Brown et al.

(1982) under Kluwe's (1982) declarative knowledge category, while placing the related tasks, strategies, and control under the procedural knowledge umbrella. Pintrich (2002) addressed the multi-faceted view of metacognition, as well, although he spoke to three types of metacognition. Strategic knowledge involves different approaches for thinking, learning, and problem solving including the ways in which students plan, monitor, and regulate their thought process (Pintrich, 2002, p. 220). In mathematics specifically, this can involve determination of an approach to a problem, checking a solution, and then evaluating and correcting where errors may have been made. Knowledge about cognitive tasks refers to the understanding that certain tasks will be more difficult than others, a view that can also be referred to as "conditional knowledge" (Pintrich, 2002, p. 221). Lastly, Pintrich (2002) wrote of self-knowledge, which includes an understanding of one's strengths and weaknesses, which can be used to help students determine how to best approach a given academic task (p. 221). If students have the ability to tap into that understanding, they are better able to approach more advanced mathematical tasks with the creative processes required for solution.

In his contributions to self-efficacy theories, Albert Bandura (1997) proposed that metacognition involves a judgment of one's thinking processes, as well as control over his or her cognition. He suggested that individuals can evaluate their thoughts for adequacy in solving problems and make any necessary adjustments in the solution process. Alongside his definition of metacognition, Bandura (1997) cautioned that metacognitive processes will only contribute to successful performance of tasks if one actually uses said processes. Metacognition is inherently tied to motivation. "People need to learn how to monitor their functioning and the effects it produces and how to structure

motivating challenges and self-incentives” (Bandura, 1997, p. 230). If this monitoring of cognitive function is a learned skill, then it is worth looking into the ways in which such processes can be taught. This can be particularly critical in the field of mathematics education, where students may struggle especially with motivation and self-efficacy. One application of the teaching and learning of metacognition could be in the realm of reflective practice.

In his work entitled *Emile*, Rousseau (2010) suggested that reflection leads to the gathering of ideas and contemplation of such ideas. He theorized that once one begins the process of thinking, one will never stop doing so. “Whoever has thought will always think, and once the understanding is practiced at reflection, it can no longer stay at rest” (Rousseau, 2010, p. 412). The implication could be that once students have been taught how to reflect about their learning, it is a strategy and practice that will stay with them throughout their educational careers. This can be particularly important with increasing demands placed on students to achieve at high levels (No Child Left Behind Act, 2001; U.S. Department of Education, 2009). Students climbing through the ranks of mathematics courses, from Algebra to Geometry to Pre-Calculus to Calculus, are expected to meet rigorous standards and often pass high-stakes assessments (Advancement Via Individual Determination, 2011). They need multiple tools upon which to draw to find success with increased expectations.

Costa and Kallick (2000) have defined being reflective as “mental wandering”; as a way to look back on where one has been in an attempt to make sense of what one has experienced (p. 61). The purpose of this mental wandering is to develop the habit of considering our experiences on a deeper level. Combining this purpose with Piaget’s

(1950) active thinking, Dewey's (1938/1997a) concept of learning through experiences, and Flavell's (1979) conscious cognition, as well as the work of others discussed here, reflection may perhaps be the next logical step once an experience has taken place.

Flavell (1976) believed that children could be taught certain metacognitive strategies to support problem solving, such as asking particular questions or making specific assertions. He encouraged researchers to determine what young students could learn with regard to this type of thinking. Metacognition and related strategies can support teaching and learning by encouraging very specific thought about one's learning. Teachers can raise students' awareness of how to generate questions, connect questions to specific knowledge, and comprehend the purpose of such questions, especially with regard to the teaching of reading comprehension (Williams & Atkins, 2009). Brown and Palincsar's (1982) two elements of metacognitive use, knowledge and control, support the learning of how to plan, monitor, and reflect on a project or scientific inquiry.

The question now is posed as to what types of experiences mathematical students should have in today's classroom. One critical component of understanding mathematical concepts is the ability to problem-solve, not just to repeat what was given in a lecture and apply the knowledge only to problems specifically related to the covered material (Tall, 1991). It is also possible for students to face academic challenges that require a level of knowledge they have not yet achieved (Pintrich, 2002). When faced with such a situation, an expert will often rely on more general strategies for learning and thinking in order to problem solve. High school mathematics students are rarely considered experts in the field, and therefore need support to build into their repertoire some of the same general strategies that experts might use. Most young mathematics students are unaware of the

cognitive processes that are required to understand mathematics at a deeper level. Yet, the process of knowledge construction of mathematical principles, which can be used to process more complex ideas, is an opportunity to engage students in the process of making unique links to their understanding of mathematical concepts (Chazan, 2000).

Conscious, active metacognitive practice is not a natural inclination for most students. As a starting point, learners must be made aware of metacognitive processes and the possible impacts on academic understanding (Schraw, 1998). This promotion of metacognition can come in multiple forms, including a blend of metacognition and constructivist theories through reflection, and more specifically, reflective assessment. Reflective practice has the potential to support learning of even the most challenging mathematical ideas.

### **Research**

Published research on reflective practices in the classroom historically seems to appear most prominently within adult educational settings, specifically the medical field (Boenink, Oderwald, De Jonge, Van Tilburg, & Smal, 2004; Hulsman, Harmsen, & Fabriek, 2009; Richardson & Maltby, 1995; Wong, Kember, Chung, & Yan, 1995) and teacher preparation field (Bain, Ballantyne, Packer, & Mills, 1999; Korthagen, 1999; Spalding & Wilson, 2002; Woodward, 1998). Some unpublished research has been completed relatively recently in classroom settings, specifically in the realm of science achievement (Bianchi, 2007; Shoop, 2006). It appears that much of the early published research involving reflection in public school settings includes a reflective piece as a component of other studies, without a focus solely on reflective assessment.

The meta-analysis carried out by Wang, Haertel, and Walberg (1993), for example, examined many of the variables in traditional school settings that can influence learning. Using information gathered from 61 research experts, 91 meta-analyses, and 179 handbook chapter and narrative reviews, the authors develop six theoretical constructs to organize their framework. These constructs include State and District Governance and Organization, Home and Community Educational Contexts, School Demographics, Culture, Climate, Policies, and Practice, Design and Delivery of Curriculum and Instruction, Classroom Practices, and finally, Student Characteristics. Average *T* scores were calculated and used to rank the constructs in terms of their influence on student learning. Second behind Student Characteristics in terms of exertion of influence was the construct of Classroom Practices. Included in this category are variables such as metacognition, cognition, classroom instruction and management, as well as others.

The authors' conclusions included the importance of attending to psychological variables, like metacognition, and the need for teaching cognitive skills in order to enrich academic learning and understanding (Wang et al., 1993). They suggested that effective lessons can be developed when teachers are aware of several factors, including students' use of learning or metacognitive strategies. They caution, however, that strategies executed poorly will likely not have the same level of success as they would if implemented with fidelity (Wang et al., 1993). It is crucial, then, to document in detail research that has been done on metacognitive practice in mathematics classrooms and to provide teachers with specifics about implementation. Additionally, the authors



acknowledged that even their primary resources may have limitations, such as shortcomings in validity.

In an effort to bring the metacognition conversation specifically to the realm of mathematics, Garofalo and Lester (1985) addressed the defining of metacognition, some of the foundational theories behind the concept, and its application to mathematical performance. The authors noted that it is important to distinguish between cognition and metacognition, especially as it applies to the study of mathematics. In a typical mathematics classroom, one will often see rote strategy usage, which involves cognition but not metacognition (Garofalo & Lester, 1985). Garofalo and Lester explained that cognition is the process or act itself, while metacognition involves making decisions about how to accomplish the process. When it comes to complex mathematical problem solving, metacognition is the coordination and management of the cognition required to complete the activity.

Garofalo and Lester (1985) presented a cognitive-metacognitive framework that focuses on four categories involved in the performance of a mathematical task, including orientation, organization, execution, and verification. They suggested such a framework can be used to help students approach mathematical learning from a metacognitive standpoint (Garofalo & Lester, 1985). It is the authors' goal that their paper stimulates conversation about metacognition and its implications for mathematical learning, stating this is only a starting point and much more research needs to be done (Garofalo & Lester, 1985). Since this work in the mid-1980s, current research on metacognition with mathematics students tends to fall into one of two major categories. Most recent studies

involve research into *how* to measure metacognition and *what* is the impact of metacognitive strategy use on learning.

**Measuring metacognition.** One of the greatest difficulties in both the study of mathematical metacognition and its application is how to measure a process that is inherently internal (Georghiades, 2004). Researchers face the dilemma of the Schrödinger's cat paradox (Schrödinger, 1980) in that attempting to measure the impacts of metacognitive practices, such as reflective assessment, actually interferes with the metacognitive process a student may be undergoing. This does not necessarily mean that attempts to measure the effects of metacognition should be abandoned, however. It simply means researchers must work to develop appropriate tools for measurement and apply them with the knowledge of possible pitfalls.

Dahl (2004) completed a qualitative study, focusing on the metacognitive awareness of 10 high-achieving high school mathematics students. Four of the students were from Denmark, while six were from England. The participants consisted of five female and five male students, all studying math at the highest level offered in their school systems. All of the students were in their final year of school, ranging in age from 17 to 20.

The participants were interviewed, mainly through unstructured focus groups, about how they learn new mathematical concepts. Dahl (2004) developed the CULTIS model as a way to organize student responses and compare explanations to theories of learning. Student responses typically fell within the six themes of the CULTIS model: Consciousness, Unconsciousness, Language, Tacit, Individual, and Social (Dahl, 2004, p.140).

Dahl (2004) came to several different conclusions during the course of her study, including the idea that learning strategies used by these students were often connected to how they were used to being taught. In other words, a student's learning history can influence how they learn new concepts. She also concluded that these particular learners have a metacognition and are successful, leading her to the question, "Can one discuss metacognition with lower-achieving students?" The author proposed that the CULTIS model could be used in two major ways. It can be utilized as a tool in the development of metacognition, as well as useful for teachers in an overview of major learning theories.

Panaoura, Philippou, and Christou (2003) piloted a study with the intention of developing an instrument measuring metacognition that would be appropriate for use with students at the elementary level, specifically for the assessment of metacognition during problem solving. The 246 students included in the study ranged in age from eight to 11. They were asked to complete a 30-item questionnaire, circling answers that best described their thought process when solving a mathematical problem. In a second part of the questionnaire, the students were asked to read non-routine problems and circle item answers that best described their thoughts while trying to think of the solution. Finally, after solving the problem, the students had to answer additional questions about their thoughts while finding the solution.

Factor analysis of the items resulted in nine factors accounting for 58.964% of the variance (Panaoura et al., 2003, p. 5). The authors grouped four of the factors under the definition of "knowledge of cognition," gathering the other five under the heading of "metacognitive regulation." Statistically significant high correlations between almost all factors were reported, which would be expected based on the correlation between the two

components of metacognition—knowledge and regulation (Brown et al., 1982). Their initial conclusions were that, with a few adjustments, the inventory they developed could become a valuable measurement tool for use in assessing metacognition in primary school mathematics students.

Wilson and Clarke (2004) designed a Multi-Method Interview (MMI) approach to assess metacognition (p. 29), defined to include awareness, evaluation, and regulation of thinking. Their extensive process for monitoring student thinking involved observation, audio and video recording, and a clinical interview. During the 90 interviews, grade six students, recruited from six different classes in Victoria, Australia, used cards with metacognitive action statements to reconstruct their thought process during problem solving. According to the authors, this multi-method approach appeared to be effective for the study of metacognitive behavior in grade six students in terms of consistency of results, but these conclusions are supported only through anecdotal evidence. No quantitative data is reported in this particular paper, although such a card-sorting technique might provide teachers with a tool that could be adapted for classroom use in conversations with students about metacognition and problem solving (Wilson & Clarke, 2004, p. 44). More work is certainly necessary to verify or dispute the authors' claims.

Jacobse and Harskamp (2012) addressed the concern that, while effective, the think-aloud metacognitive protocol can be time consuming and very complex. In a classroom of 25 to 30 or more students, it would be nearly impossible for a teacher to efficiently utilize such a strategy. According to the authors, self-report questionnaires may provide an alternative process, but evidence suggests a lack of convergence between these types of questionnaires and the think-aloud, perhaps due to memory distortion. As

such, Jacobse and Harskamp (2012) proposed a more practical metacognitive measure, comparing this new instrument with think-aloud scores. Thirty-nine randomly selected fifth graders participated in the study, using word problems with an adequate level of difficulty to allow for a diverse range of metacognitive strategies. The instrument being evaluated in the study was called the “on-line prediction-visualization-postdiction” instrument (VisA), with the authors assessing convergence between VisA, self-report questionnaires, and think-aloud instruments. Ultimately, the question was, “Can the [VisA] predict problem solving on an independent mathematical word problem test just as well as a think-aloud measure?” (p. 137).

Students in this study scored low on all metacognitive measures, leaving the authors to propose that metacognitive strategy use is still in the developing stages in fifth grade, although they gave no other studies or evidence to back up that particular conclusion. The results of the study did match what the authors seemed to have anticipated, based on theory and other studies. The think-aloud and VisA showed higher correlations with performance than the self-report questionnaires, with think-aloud accounting for 33% of the variance and VisA accounting for 23%. According to this specific study, the self-report questionnaires also had no convergence with online measures, suggesting they are not a good substitute for think-aloud or VisA strategies. Only a moderate correlation between on-line measures was found. Jacobse and Harskamp (2012) concluded that self-report questionnaires are a better measure of metacognitive knowledge than of metacognition itself. Where a think-aloud strategy may be too time-consuming for a full classroom of students, the combination of prediction, visualization and postdiction judgments (VisA) could be a more efficient substitute.

Interviews and inventories may provide researchers with empirical information about students and their use of metacognitive strategies, giving us valuable tools for understanding how metacognition supports mathematical learning. The reality of the classroom, however, often calls for less time-consuming procedures to approach assessment of thinking. While the research in which more extensive procedures can be utilized is extremely important, there is also a need for what might be considered more classroom-friendly versions of metacognitive practice and measurement.

Researchers and educators explored the impacts of alternatives to the typical direct instruction, homework, and test pattern of most secondary mathematics classrooms. McIntosh (1997) wrote of the power of interviewing students using a specific set of metacognitive questions to help the instructor understand learner processing. Referring to a selection of anecdotal experiences and strategies utilized in her own classroom, McIntosh suggested that verbal interviews are one method that could be utilized, but have the disadvantage of being time-intensive, similar to the MMI approach (Wilson & Clarke, 2004). An alternative to the verbal interview is the use of learning logs or a similar writing form, which provides students with a forum for refinement and clarification of their thought processes (McIntosh & Draper, 2001). Techniques such as the use of learning logs or other similar reflective practices can be used in future research surrounding metacognition in mathematics classrooms.

One example of such research is a recent study (Bond & Ellis, 2013) analyzing the effects of reflective assessment on mathematics achievement among 141 Grade 5 and Grade 6 students. Students were randomly assigned to three conditions—reflective assessment, non-reflective review, and a control group. Students in reflective assessment

experimental group used a combination of written and verbal strategies to process their own learning, in the form of written “I Learned” statements and verbal “Thinking Aloud” strategies (Ellis, 2001). Such procedures are not as time-intensive as a simple interview process or MMI approach, perhaps making them more appropriate for the everyday classroom constraints.

Bond and Ellis (2013) found that initially students using reflective strategies showed statistically significant mathematical achievement over those who did not on both the posttests and retention tests. The authors also reported medium to large effect sizes, which indicated the practical significance of such strategies within a real classroom. Random assignment of student participants, as well as teachers to groups, supported the strength of the findings through balanced groups and instruction consistency. Use of a suburban, middle class school sample does limit the generalizability of the study. There is enough evidence of the effectiveness of reflective practice in the study, however, to support continued research on this particular topic. The authors suggested further research of the use of such strategies in diverse student populations.

**Effects of metacognitive practice at the primary level.** Metacognitive knowledge may actually be present quite early in students’ mathematical learning. In a study involving 39 second grade students from the Munich International School, Carr et al. (1994) completed interviews with individual students about their strategy use in mathematics. Through statistical analyses of intercorrelations, it was found that correct internal strategy use correlated with metacognition. The authors concluded “even second graders possess metacognitive knowledge about mathematics strategies and that this knowledge of mathematics strategies is affected by prior correct strategy use” (p. 591).

The authors claimed that younger students can benefit from knowledge about mathematical strategies. They reported a statistically significant increase in the prevalence of internal strategy use from September to January from 23% to 36% of the total number of problems. They also noted a significant increase in correct internal strategy use over that same time frame.

It is important to note that apparent relationships between external strategy use and metacognition are not as strongly correlated as those for internal strategy use and metacognition. Carr et al. (1994) also acknowledged that some previous studies (Siegler, 1989; Siegler & Shrager, 1984) arrived at different conclusions about young mathematics students, but argued that this could be due to the different aspects and levels of metacognition that were examined in prior studies. Carr et al. (1994) also noted that their data does not clarify which types of metacognition are most critical for correct strategy use.

Desoete (2007) found that, while metacognitive skills may not necessarily come naturally to students, they are teachable. In her study involving 33 Grade 3 and Grade 4 students in Belgium, Desoete examined teacher rating of mathematics performance and metacognition and student completion of a mathematics test in which students were required to think aloud. Statistical analysis of teacher ratings and mathematical performance determined that metacognition accounted for 22.2% the mathematical performances of these grade school children. Through her literature review and previous work, the author concluded, "...metacognitive training improved pupil performance in mathematical problem solving and was found to have a sustained effect on mathematical problem solving" (Desoete, 2007, p. 718).



These skills had to be explicitly taught, according to Desoete (2007), supporting previous findings that it cannot be assumed such methods are developed simply through the experience of mathematics (Desoete, Roeyers, & De Clercq, 2003). There are some limitations to the study, including use of a small, non-random sample and the fact that analyses were done based on only two teachers. The author did not specifically address the ways in which metacognitive skills should be taught nor did she present data in this particular study to support her conclusions about the need for teaching metacognitive strategies. In spite of these limitations, the author did find a tentative link between metacognition, as rated by the teachers, and pupil performance in mathematics. The researchers used self-report questionnaires, students indicating their own levels of metacognitive prediction, planning, monitoring, and evaluation skills on a seven-point Likert scale. Cronbach's alphas reported for the instruments support their internal reliability. Desoete also brought to the conversation an important question—can mathematical metacognitive skills develop naturally or must they be explicitly taught to students?

Naglieri and Johnson (2000) chose to investigate the impact of cognitive strategy instruction with regard to a very specific population of students—those with learning disabilities and mild mental impairments. The 19 students involved in the study were given mathematical tasks on worksheets and were encouraged to verbalize their processes, with the ultimate goal being to improve the students' application of planning and reflective methods.

Students initially participated in baseline activities using mathematics worksheets, without receiving any feedback. In the intervention phase, students completed a similar

mathematics worksheet and then participated in a discussion about effective strategies before completing a second worksheet. Discussions were designed to encourage conversation about self-reflection and an understanding of the need for planning and use of effective strategies. Due to the very specific population being studied, results differed for students with various cognitive weaknesses. The authors reported Cohen's *d* effect sizes that are large for students with planning weaknesses (effect size of 1.4), but those with weaknesses in attention (effect size of .3) and successive processing (effect size of .4) saw smaller effect sizes. Performance of students with simultaneous processing weaknesses actually deteriorated. Naglieri and Johnson (2000) reported that students with deficits in planning may benefit from cognitive strategy instruction, but recognize that the small sample size and specific population of students limits the generalizability of the results.

**Effects of metacognitive practice at the secondary level.** Lester et al. (1989) investigated the role of metacognition in the mathematical problem solving of two classes of seventh graders, specifically with regard to how their metacognitive beliefs and processes impacted problem solving behavior and whether such processes could be taught. The "regular" class of 28 students and "advanced" class of 37 students participated in extensive instruction over 14 weeks intended to build students' awareness and control over their cognitive processes and performance. The researchers began with a combination of pretests, interviews, and observations to guide the instructional practices, which incorporated aspects of Brown and Palincsar's (1982) self-control training and Charles and Lester's (1982, 1984) teaching strategy for mathematical problem solving. After the instructional phase was completed, data, in the form of posttests, clinical

interviews, observations of problem-solving sessions, student work, and videotapes of classroom instruction, was analyzed.

This particular study resulted in a significant amount of data that even the authors acknowledged to be “overwhelming” (Lester et al., 1989, p. 17). Most of the data were qualitative, in the form of interviews and observations. Pretest and posttest results provided a quantitative component to the data, indicating an overall gain in total scores for both the regular and advanced classes. The authors reported only the raw mean scores, however, without indicating statistical significance or nonsignificance. They noted that for individual students, some posttest scores were actually lower than pretest scores, implying that metacognitive instruction may actually have been detrimental to some students’ mathematical achievement. It was suggested that the new techniques may have interfered with prior methods that worked well for those students previously.

While their work is rich in terms of the amount of qualitative data, Lester et al. (1989) could only make tentative observations about the impact of instruction on student problem-solving ability and metacognitive awareness. Individual interviews led to insight about particular students, but no generalizations can be made from their reported results. Lester et al. (1989) acknowledged that they did not gain significant or specific information regarding which activities were effective or ineffective. They did see this study as a starting point for continued research with this age of students and the role of metacognition in mathematical problem solving. It is clear that much more work needs to be done.

Kramarski and Zodan (2008) claimed that current research has not clearly determined possible benefits or downsides to combining metacognitive approaches to

learning mathematics. The purpose of their study was to investigate the impact of three metacognitive instructional approaches on mathematical reasoning, as well as analyzing the effects of these approaches on conceptual errors and metacognitive knowledge.

Students participating in the study were 115 ninth-graders from a junior high in Israel.

Classes were randomly assigned to the different metacognitive approaches, with one class being assigned as the control. No significant differences in mathematical knowledge existed between the classes, as demonstrated by school testing on eighth grade topics.

The different metacognitive approaches included the diagnostic errors approach (DIA), IMPROVE approach (IMP), and combination of DIA and IMP, as well as the control group (Kramarski & Zodan, 2008). With the control approach, students were not explicitly taught metacognitive strategies, but instead focused solely on learning the material individually or in groups. DIA approach involved a direct analysis and discussion of conceptual errors, meant to encourage reflection. IMP included self-questioning during certain mathematical activities. The combined group utilized both metacognitive approaches. Analysis using ANCOVA and further post hoc tests indicated significant differences among the groups regarding procedural skills, with combined DIA and IMP students outperforming the DIA group, which outperformed the IMP group. All groups outperformed the control. With regard to problem solving skills, the results were similar, with all groups outperforming the control. DIA and IMP groups outperformed IMP, which outperformed DIA. Overall, the authors' conclusions were that students using both DIA and IMP strategies achieved more positive outcomes with regard to mathematical reasoning than those who only used one or the other.

Perhaps one of the most critical statements made by Kramarski and Zodan (2008) was their assertion that this study adds to the case that “metacognition is teachable” (p. 147). They also asserted that the use of more than one approach is more beneficial to students than perhaps only focusing on one strategy. The authors “call for a metacognitive culture” (p. 148), with a focus on the acceptability of errors and their dissection as a path to understanding of the material.

In a study involving 140 randomly selected students, ranging in age from 17 to 20 years, from two randomly chosen high schools in South Africa, Maqsood (1997) examined possible relationships between the use of metacognitive strategies and academic performance on mathematics tests. The Swanson Metacognitive Questionnaire (Swanson, 1990) was adapted from previously developed measures (Kreutzer, Leonard, & Flavell, 1975; Myers & Paris, 1978) and then modified for the specific purpose of work with mathematics. Responses by the students to the items were quantified according to a one to five ranking outlined by Kreutzer et al. (1975) to classify the responders as high-metacognitive performers and low-metacognitive performers. Two independent judges scored the responses, with inter-rater reliability of 93%. These scores were then combined with students’ scores from the Raven’s Standard Progressive Matrices (Raven, 1985), designed to measure general nonverbal reasoning ability, to produce four classifications of students: high general ability-high metacognition (HGA/HM), high general ability-low metacognition (HGA/LM), low general ability-high metacognition (LGA/HM), and low general ability-low metacognition (LGA/LM). An objective-type mathematics achievement test, developed by two mathematics teachers, was also used. Each of these three instruments had relatively high test-retest reliability coefficients: the

Swanson Metacognitive Questionnaire (SMQ) ( $r = 0.79$ ), the Raven's Standard Progressive Matrices (RSPM) ( $r = 0.83$ ), and the mathematics achievement test ( $r = 0.88$ ).

Maqsud (1997) completed a 2 (high vs low general ability)  $\times$  2 (high vs low metacognitive ability)  $\times$  2 (males vs females) ANOVA on mathematical performance, resulting in significant effects for general ability, metacognitive ability, and gender. Interaction effects were not found to be significant, however. The key finding of this study, with regard to the focus of this paper, was that of statistically significant differences in mathematics scores between students with high metacognitive ability and those with low metacognitive ability, as classified by the SMQ. When high general ability is constant, students with high metacognitive ability scored significantly higher than those with low metacognitive ability,  $t(68) = 3.10, p < 0.01$ . Results were also statistically significant when low general ability is constant, with students with high metacognitive ability mathematically outperforming those with low metacognitive ability,  $t(68) = 5.25, p < 0.001$ . Perhaps most interesting was the result that students with high general ability and high metacognitive ability had the highest mathematical performance, while performance was lower for those with high general ability and low metacognitive ability. The author concluded from this finding that metacognitive abilities are positively associated with success regarding mathematical achievement scores, but states there is still a need for exploring possible cause and effect relationships.

**Effects of metacognitive practice at the university level.** Rosenthal (1995) addressed the concept of reflective assessment in mathematics at the university level in a paper encouraging teachers to experiment with alternative teaching strategies. While not

specifically called “reflective assessment,” the “Minute Paper” (Wilson, 1986) and other metacognitive exercises (Angelo, 1995) were suggested as possible methods for helping university math students consider their thought processes and provide feedback for their professors. Rosenthal (1995) recommended instructing students to spend a few minutes at the end of class writing about what they felt was the key topic, most confusing concept, and what they most wanted to know more about from the lecture. He concluded that college mathematics course lectures could be supplemented with strategies that encourage active processing of course material (Rosenthal, 1995). This promotional paper includes no insight as to how such strategies have been developed or tested, however.

Wilson (1986) discussed the “Minute Paper” as part of the results of a three-year research study designed to analyze the impact of telling college faculty members about good teaching practices. Students were asked to complete a questionnaire focused on five factors developed through an item-analysis study (Hildebrand, Wilson, & Dienst, 1971). These five teaching factors were identified as organization and clarity, analysis and synthesis, teacher-student interaction, teacher-group interaction, and dynamism and enthusiasm. Teachers who received high ratings were then interviewed to determine possible reasons why students had rated them so highly. One particular interview introduced the “Minute Paper” strategy. Wilson (1986) suggested this as one example of good teaching practice that could be introduced to college faculty, but he does not do any analysis of the practice itself or its impact on students beyond the anecdotal interview with the highly rated teacher. This reflective strategy is one that could be explored further, but was suggested with no empirical backing.

Hudesman et al. (2013) recognized that a significant number of students entering college are not considered “academically ready,” leading many of these students to need to take developmental courses. The authors of this study viewed formative assessment as a powerful intervention that can improve academic performance. Focusing on developmental math classes, a very specific model was implemented. The Enhanced Formative Assessment Program (EFAP) with a Self-Regulated Learning (SRL) component became the foundation for the two summer and two academic-year studies. In one of the studies, students were randomly assigned to the comparative group or EFAP-SRL cohort. Specially formatted quizzes, with a metacognitive judgment component, were used, along with a self-reflection and mastery learning form. The studies produced some interesting results, with  $\chi^2$  indicating statistically significant differences ( $p < .05$ ) between the comparison and experimental groups with regard to passing the developmental course and pass rates on the COMPASS test. While the results were promising, some instructors indicated concern about the additional time necessary for specialized quizzes and reflection, especially in light of an already full curriculum.

**Understanding through conceptual writing.** In addition to the effects of metacognitive components to writing, researchers explored the specific impacts of writing about conceptual themes. Bangert-Drowns, Hurley, and Wilkinson (2004) completed a meta-analysis of 48 writing-to-learn programs. They found that programs including specific reflection prompts are especially effective and that students who are unacquainted with a specific topic may receive more benefit from the writing process than those who are familiar with the concept.



Rivard and Straw (2000), in a quasi-experimental study involving 43 eighth grade science students in Canada, analyzed the role of talk and writing for learning. The researchers acknowledged that the small sample size limited their ability to draw certain conclusions, but through qualitative analysis of the student writings, they suggested that writing plays a significant role in the organization of students' thoughts and ideas with regard to specific science concepts, a necessary step for construction of conceptual understanding.

In a study involving 104 middle school students at the International College in Beirut, Jurdak and Zein (1998) explored the effect of journal writing on conceptual understanding, procedural knowledge, problem solving, mathematics school achievement, and mathematical communication. Results of the MANCOVA revealed a significant main effect of the treatment (Hotelling's  $T^2(6,92) = 18.32, p < .00$ ). Consideration of univariate  $F$ 's indicated the mean scores of the journaling group to be significantly higher than the control for conceptual understanding, procedural knowledge, and mathematical communication. No statistically significant differences were found for problem solving, school mathematics achievement, and attitudes toward mathematics. The authors attributed the positive effect of writing on conceptual understanding to the close relationship between language and concepts.

Exploring the impact of writing-to-learn activities on conceptual understanding in calculus, Porter and Masingila (2000) assigned one university introductory calculus course to the treatment group and another to the comparison group. The treatment group participated in a variety of writing-to-learn activities, both during and outside of class, which included having students write about specific course ideas, concepts, and

procedures in their own words. The comparison group also focused on activities regarding specific concepts, but the activities did not involve writing. Feedback from the instructor was given to both groups. MANOVA for four different exams was completed. For only one of the four exams, a statistically significant difference was found between the treatment and comparison groups,  $F(2,30) = 3.87, p = .03$ . The authors asserted that the benefit in writing about calculus concepts may not be in the writing itself, but in the time spent thinking about and communicating mathematical ideas.

**Opportunities for additional research.** With the increased popularity of college preparatory programs such as Advanced Placement, International Baccalaureate, and Cambridge International Examinations, more students have the opportunity to be exposed to rigorous mathematics courses in high school than in the past. Some state and school district policies have also influenced the number of students enrolled in advanced courses (Revised Code of Washington, n.d.). Currently expanding programs like Advancement Via Individual Determination (AVID), which emphasize rigor such as that offered in advanced mathematics courses, rely on specific instructional strategies, including reflection, to help support students in the more than 4,500 sites where the program has been implemented (AVID, 2011). As programs like this one gain popularity and claim to be research-based (AVID, 2011), it is important that additional research be done to either support or refute their claims.

With more programs and policies that encourage students to enroll in higher levels of mathematics, the specific field of advanced high school mathematics seems ripe for the investigation of more structured metacognitive instruction and practice. There is a need for more research on the use of metacognitive strategies to support student learning

of more advanced mathematical curricula. More specifically, the realm of advanced high school mathematics provides a valuable opportunity for the study and application of reflective assessment.

### **Conclusion**

Over the past few decades, the accountability conversation in education increased in intensity. The field of mathematics education seems to be a particularly critical component in that conversation. Many young students struggle to take mathematical problem solving and procedures to new levels and new applications beyond the few examples they see during typical classroom instruction. Educators need additional tools in their instructional toolboxes to help support mathematical learning that utilizes true problem solving skills. Metacognitive understanding, teaching, and practice in the form of reflective practice could perhaps hold the key to increased mathematical achievement.

As assessment becomes even more central to this same discussion surrounding accountability and improvement of mathematics education (Senk, Beckmann, & Thompson, 1997), it seems important to consider possible alternatives (Marzano, 2009) to the assessments most often used in traditional high school mathematics classrooms (Watt, 2005). When looking at alternatives, one impactful curriculum change in the field of mathematics could be more use of assessment *for* learning (Wiggins & McTighe, 1998). Metacognitive and reflective theories (Bandura, 1997; Brown, 1994; Brown et al., 1982; Dewey, 1938/1997a; Dewey, 1910/1997b; Flavell, 1979; Piaget, 1954; Pintrich, 2002) are a solid theoretical foundation for one particular form of assessment for learning, that of reflective assessment. It might be suggested that based on these theories, and much of the promising research that has already been completed, reflective

assessment specifically could be a valuable addition to mathematics curriculum at all levels.

While research in primary, secondary, and university mathematics and with special populations is showing some promise in the influence of reflective practice on learning, more work certainly needs to be done. One “root meaning of ‘to educate’ is ‘to draw out’ and...the teacher’s task is not to fill the student with facts but to evoke the truth the student holds within” (Palmer, 1983/1993). It is important to continue to explore reflective practice as one possible method for “drawing out” student understanding from within. With the influence of No Child Left Behind (2001), the Race to the Top Assessment Program (U.S. Department of Education, 2009), and a renewed emphasis on assessment, perhaps reflective assessment is one way to evoke said truth. As educators find new ways to increase student achievement, it is important to remember that reflection can allow students to view their own learning and can lead to a more solid foundation of problem solving skills as students become aware of the cognition behind such procedures.

Metacognitive practices like reflection, and perhaps more specifically reflective writing, could be the tools that help lead to deeper student understanding in advanced mathematics. Responsibility for metacognitive and reflective practice cannot rest solely on the shoulders of the classroom instructor, however. The baton of such action must be passed to the students themselves. Metacognition and reflection are about the self. Therefore students must be taught how to apply such practices in their own learning, although this may be an incredible challenge in an age of teaching with a strong focus on academic standards.

## **Chapter Three**

### **Methodology**

#### **Research Design**

The primary research question was as follows: Does the practice of reflective writing have a statistically significant impact on the mathematics achievement of Advanced Placement (AP®) Calculus students? The null hypothesis was that the practice of reflective writing does not have an impact on the mathematics achievement of AP Calculus students. This question was designed to explore a possible causal relationship between reflective practice and achievement in advanced mathematics. As such, the most appropriate research design would be experimental.

Within the context of a school setting involving intact classrooms, random assignment cannot occur. Therefore, the research design was a pre- and posttest control group quasi-experimental study (see Table 1). In addition to the primary research question, the following descriptive and correlational questions were also posed: What themes emerge from the reflective journal entries of the AP Calculus students? What is the correlation between the students' number of extracted themes from the reflective journal entries, the sentiment value of those themes, and their scores on the posttest? The null hypothesis for the second question was that a positive correlation does not exist between the number of extracted themes, the sentiment values, and a students' score on the posttest.

Table 1

*Pretest-Posttest Control-Group Design*

Group	Pretest	Intervention	Posttest
N <sub>1</sub>	O	X	O
N <sub>2</sub>	O		O

*Note.* N<sub>1</sub> = non-randomly assigned experimental group, N<sub>2</sub> = non-randomly assigned control group

The school was selected for convenience with each of the two Advanced Placement (AP®) Calculus classes being assigned to either the experimental and control groups. The independent variable was the use of metacognitive writing practiced only by the experimental group.

Students in this group used three metacognitive writing strategies including “I Learned” statements (Ellis, 2001; Simon, Howe, & Kirschenbaum, 1972), “Clear and Unclear Windows” (Ellis, 2001), and the “Minute Paper” (Wilson, 1986). The dependent variable was student achievement as measured by mathematics assessment scores. All students participated in the regular instruction, practice, and assessment as planned by their instructor. In addition to measurement of achievement through the math assessment scores, content analysis was completed on a selection of student metacognitive writings from the experimental group.

**Setting.** This study involved Advanced Placement (AP®) Calculus students at a comprehensive suburban high school in Washington State (see Appendix A). The school serves a local community of fairly low socio-economic status, with 60.5% of the students on free or reduced-price lunch at the beginning of the study. Enrollment of the school for

students in grades nine through 12 was 1,521. The ethnic breakdown at the high school included 11% Asian, 15% African American, 28% Hispanic, 9% Multi-Racial, 1% Native American, 8% Pacific Islander, and 28% White. Faculty members at the high school numbered 81 with an administrative team of four. The high school had a graduation rate of 80.1%. The class schedule at this high school was a traditional six-period day with an advisory period three days a week. The class periods met for 54 minutes on Mondays and Fridays. They met for 50 minutes the remaining three days of the week with a 25-minute advisory between first and second periods.

**Participants.** Two AP® Calculus classes participated in this study putting the total number of student participants at 42. Of the participants, three were in their junior year and the remaining 39 were in their senior year with four repeating the course. Of the 42 participants, 21 were female and 21 were male. Nine of the students were part of the English Language Learners program, one had a 504 plan, one qualified for Student Support Services, 12 qualified as Highly Capable, 17 were on free or reduced-price lunch, and 23 were part of the Advancement Via Individual Determination (AVID) program. Students in the AVID program receive additional support in college preparation and readiness and are required to enroll in at least one advanced course each year. AVID students are typically those from underrepresented populations and many will be the first in their families to attend college.

The ethnic breakdown of the participants included 26% Asian, 14% African American, 19% Hispanic, 5% Multi-Racial, 14% Pacific Islander, and 21% White. Of the students enrolled in the course for first semester, 31% received As, 40% received Bs, 24% received Cs, and 5% received Fs. This district does not recognize the letter grade of

D as passing nor does it utilize a plus and minus system. This results in a grading scale consisting only of A, B, C, F, or Incomplete. Absences for first semester ranged widely, from zero absences to 24. The average number of absences per student was eight.

**Sampling.** The sample was chosen for convenience purposes due to the researcher's position at this particular high school allowing access to these specific classes. One intact class section was randomly assigned as the experimental group. The remaining class section was utilized as the control group. The researcher presented to the classroom teacher detailed information and instruction about the metacognitive writing strategies to be used (Ellis, 2001; Simon, Howe, & Kirschenbaum, 1972; Wilson, 1986) and how students should use each writing practice (see Appendix B). The classroom teacher presented this same information to the experimental class students, as well as guided them to complete the reflective writing activity each day at the end of class period. The experimental group only completed the writing activity and did not complete the control group's closing activity problems. Students in the control group were instructed by their teacher to complete their closing activity problems in their notebooks receiving no instruction in metacognitive writing strategies. The control group did not participate in any reflective writing practice in this class over the course of the study. Reflective writing practice was implemented as a part of the curriculum for AP Calculus in line with the district focus on literacy and on exit task practices. General results received from this study will likely be used to inform the use of exit tasks in the form of reflective writing throughout the high school.



**Independent variable.**

*Experimental group.* The independent variable was the use of reflective writing at the end of each class period for the experimental group. There were three types of reflective writing that were used. The instructor rotated through the various strategies so that students utilized all three several times throughout the course of the study. Students were given a composition notebook in which to complete their reflective writings. These notebooks were separate from homework or classwork activities. The three reflective writing activities are described below.

*The Minute Paper.* The Minute Paper, as described by Wilson (1986), involved students answering two very specific questions. Students responded in their notebooks to “What is the most significant thing you learned today?” and “What question is uppermost in your mind at the end of this class session?” (Wilson, 1986, p. 199). The Minute Paper was selected for use to bring in a college or university strategy. As AP Calculus is designed to be a college-level course, it was appropriate to include a writing strategy recognized for its use at the university level.

*I Learned Statement.* This particular strategy was approached in the way one would expect. At the end of a class period, students completed a statement in their notebooks about what they learned personally during that class period (Ellis, 2001; Simon et al., 1972). While this may overlap a bit with the Minute Paper, it allowed freedom for students to describe more of what they learned than just the most “significant” thing. “I Learned” statements were utilized in a previous study with fifth and sixth grade math students (Bond, 2003; Bond & Ellis, 2013) in combination with an additional “think aloud” or “Talk About It” strategy (Ellis, 2001).

*Clear and Unclear Windows.* In this strategy, students divided a page of their notebooks into two parts. On one side they listed what was clear about the day's lesson. On the other they listed out what was unclear (Ellis, 2001). This self-assessment strategy could be used to expose misconceptions and encourage students to dig past "parroting back superficial information" (Ellis, 2001, p. 74).

The decision was made to utilize a variety of written strategies for the purposes of this study rather than one single strategy. These three strategies, while similar in nature, vary in ways that may help students think slightly differently about their metacognitive processes. Each had a slightly different emphasis allowing for some variety in the student process to avoid boredom or a novelty effect (Gall, Gall, & Borg, 2007). While many previous studies also include an oral reflective component (Bond, 2003; Jacobse & Harskamp, 2012; Wilson & Clarke, 2004), such as a think aloud strategy of some form, this study intended to focus solely on the impact of written strategies on student achievement. McIntosh and Draper (2001) suggested that written journaling could be a more time-efficient alternative to think aloud strategies.

***Control group.*** Since the control group did not participate in the reflective writing intervention, these students utilized their composition notebooks to complete an end-of-class problem related to that day's lesson. During the last five minutes of class, students were given the problem to work through in their notebooks. The AP Calculus teacher, based on what was covered in class that day, developed and assigned these problems. The problems were strictly mathematical in nature and did not involve any written reflection.

**Dependent variable.**

***Instrumentation.*** The measure of the dependent variable, mathematics achievement, was a portion of the non-calculator multiple-choice component of the released 1998 and 2008 AP® Calculus AB Examinations (The College Board, 1999, 2009). The pretest was used as a covariate to statistically adjust for variability, since random assignment was not used. The posttest was used to determine if statistically significant differences existed between the experimental and control groups after the intervention occurred. The classroom teacher administered the assessments utilizing only valid and reliable released 1998 and 2008 AP examination multiple-choice questions, and the researcher graded the pre- and posttests as outlined by the College Board. This objective grading process involved labeling a student's answers on the bubble answer sheet as correct or incorrect. The non-calculator multiple-choice portion of the released 1998 AP Calculus AB examination was used for the pretest, and the released 2008 AP Calculus AB examination was used for the posttest. These particular multiple choice exams were the two most recently released by the College Board. As the reliability of both exams was at an acceptable level, the exams were randomly assigned to serve as the pretest and posttest. Students were given a 54-minute class period to attempt all 28 questions. As the test was designed to be 28 questions in 55 minutes, it was possible some students did not complete all pre- and posttest questions. Both the control and experimental classes were under the same time constraint.

***Instrument reliability.*** Reliability data provided by the College Board (Educational Testing Service, 1998, 2008) indicated a satisfactory level of reliability for the multiple-choice portion of the 1998 and 2008 AP Calculus AB exam. The reported

reliability coefficient is 0.813 for the non-calculator portion of the 1998 exam and 0.845 and 0.846 for the two different forms (Q and R) of the exam in 2008. Vogt (2005) suggested reliability coefficients about 0.70 indicate an acceptable level of reliability.

***Instrument validity.*** College Board reported statistically significant predictive validity for students who score three or higher on the AP Calculus exam with regard to first year college GPAs and retention rates as well as institutional selectivity (Mattern, Shaw, & Xiong, 2009). The AP Calculus assessment used for measurement of the dependent variable appears to have high face and content validity as the College Board process for development of such exams involves university and subject-area experts.

### **Procedure**

The pretest was given at the beginning of the data collection period. Both classes took the pretest. The classroom teacher received instruction with regard to the writing strategies as well as follow-up instruction from the researcher when there were lingering questions. The teacher instructed the experimental class about the use of reflective writing the day after the pretest with demonstration of the three types of metacognitive writing strategies and a guide for student reference that was included in their notebooks to remind students of the processes (see Appendix B). The three strategies were used throughout the remaining weeks of the study with students writing for the last five minutes or so of each class period. To ensure the fidelity of the intervention, the teacher randomly selected student journals weekly for the researcher to read through for proper use of the strategies. After the first two weeks of writing, the researcher noticed the written entries did not appear to have an appropriate level of depth. The researcher worked with the teacher to provide additional reminders for students to write as much as

possible during the time given. Later analysis of the journals during the data collection period indicated some level of improvement in the amount and depth of student writing.

The control group class completed an alternative closing activity during the last five minutes of class in the form of a math problem related to the topic for that class period as selected by the teacher. During the final week of the study, both classes took the posttest. The entire study spanned about six school weeks with students participating in the experimental and control activities for 29 class periods.

### **Data Analysis**

**Research question 1.** Does metacognitive writing increase the level of mathematical understanding for Advanced Placement (AP®) Calculus students as shown through measurements of achievement? The null hypothesis was that metacognitive writing has no impact on the mathematical understanding of AP Calculus students. The goal of this particular research question was to determine if various forms of reflective writing practice impact achievement in advanced mathematics. Inferential statistics were the most applicable. An analysis of covariance (ANCOVA) allowed comparison of the means of the experimental and control groups for the posttest with the pretest as the covariate. The means being compared were those of the mathematical achievement of the two groups as measured by the summative assessments given at the end of the intervention timeframe.

**Research question 2.** What themes emerge from the reflective journal entries of the AP Calculus students? In addition to examination of student achievement scores, a content analysis was completed on a selection of student reflective writings. Eleven notebooks were chosen based on the appearance of detailed writings and analyzed for

specific content and themes within the student writing. This selection of notebooks represented half of the students in the experimental group. Theme extraction was completed utilizing Semantria® text analysis software with predetermined themes based on the calculus content addressed during the timeframe of the study.

**Research question 3.** What is the correlation between the students' number of extracted themes from the reflective journal entries, the sentiment value of those themes, and their scores on the posttest? The null hypothesis was that there was no positive correlation between the number of extracted themes, the sentiment values of the themes, and a students' score on the posttest. Bivariate correlation was conducted to determine if there was a correlation between student achievement scores on the posttest and number of specific extracted themes in the writing samples. Pearson's  $r$  was the most appropriate correlation coefficient as the data was at the interval level (Field, 2009).

## Chapter Four

### Results

#### Overview

The purpose of this study was to examine the effects of metacognitive writing on student achievement in Advanced Placement (AP®) calculus. Three research questions were posed:

1. Does metacognitive writing increase the level of mathematical understanding for Advanced Placement (AP®) Calculus students as shown through measurements of achievement? The null hypothesis is that metacognitive writing has no impact on the mathematical understanding of AP Calculus students.
2. What themes emerge from the reflective journal entries of the AP Calculus students?
3. What is the correlation between the students' number of extracted themes from the reflective journal entries, the sentiment value of those themes, and their scores on the posttest? The null hypothesis is that there is no positive correlation between the number of extracted themes, the sentiment value of the themes, and a students' score on the posttest.

What follows is the analysis of the data produced by students at the end of the six-week quasi-experimental study. Detailed information about the participants will be given first, followed by the reporting of descriptive statistics. Statistical data regarding each of the three research questions will be presented, as well as the interpretation of the results. Finally, significant results will be summarized.

## Study Participants

This study was completed in AP Calculus classes at a comprehensive suburban high school in Washington State. Tables 2 through 4 summarize specific demographic information regarding the students in the experimental and control groups who completed the entire study. The experimental group experienced attrition of three students from the original sample of 24. A breakdown of the groups by gender and by grade level is shown in Table 2. Racial demographics are shown in Table 3 while student qualification for or participation in special programs at the school is shown in Table 4.

Nineteen female and 20 male students participated in the complete study. Of the participants, the large majority was in 12<sup>th</sup> grade, as one would expect for most high school AP Calculus courses. Two of the students were in 11<sup>th</sup> grade while 37 were in 12<sup>th</sup> grade. Eleven students identified as Asian, four as African American, eight as Hispanic, one as Multi-Racial, six as Pacific Islander, and nine as White. Sixteen of the students qualified for free or reduced price lunch, 20 participated in the Advancement Via Individual Determination (AVID) program, nine qualified for English Language Learner (ELL) services, one had a 504 plan, one qualified for Student Support Services (SSS), and 11 qualified as Highly Capable.



Table 2

*Gender, Grade Level, and Socioeconomic Designation by Group*

Group	Female	Male	11 <sup>th</sup> Grade	12 <sup>th</sup> Grade
Experimental	10	11	1	20
Control	9	9	1	17
Total	19	20	2	37

Table 3

*Ethnicity by Group*

Group	Asian	African American	Hispanic	Multi- Racial	Pacific Islander	White
Experimental	4	1	5	0	6	5
Control	7	3	3	1	0	4
Total	11	4	8	1	6	9

Table 4

*Special Programs by Group*

Group	Free/Reduced Price Lunch	AVID	ELL	504	SSS	Highly Capable
Experimental	10	12	4	1	0	6
Control	6	8	5	0	1	5
Total	16	20	9	1	1	11

## Results

**Descriptive statistics.** An analysis of covariance (ANCOVA) was performed to assess the impact of metacognitive writing activities on student achievement for the experimental group in comparison to the control group who completed no such writing. The independent variable was the use of reflective writing at one level while the dependent variable was the score on the posttest, the non-calculator multiple choice portion of the released 2008 AP Calculus AB examination. The pretest score, from the non-calculator multiple choice portion of the 1998 AP Calculus AB examination, was the covariate. Descriptive statistics for the pretest scores for the experimental group ( $N = 21$ ;  $M = 5.10$ ;  $SD = 3.13$ ) and control group ( $N = 18$ ;  $M = 7.44$ ;  $SD = 2.57$ ) are displayed in Table 5, while Table 6 displays the descriptive statistics calculated from the posttest scores for the experimental group ( $N = 21$ ;  $M = 4.05$ ;  $SD = 2.36$ ) and control group ( $N = 18$ ;  $M = 7.44$ ;  $SD = 3.24$ ).

Table 5

*Descriptive Statistics: Pretest*

Group	$N$	Mean	$SD$	Range	Skewness	Kurtosis
Experimental	21	5.10	3.13	14	.78	2.21
Control	18	7.44	2.57	9	.56	.05

Table 6

*Descriptive Statistics: Posttest*

Group	<i>N</i>	Mean	<i>SD</i>	Range	Skewness	Kurtosis
Experimental	21	4.05	2.36	8	-.01	-.69
Control	18	7.44	3.24	12	.52	-.17

The value of kurtosis displayed in Table 5 for the pretest with the experimental group does raise concerns for normality (Field, 2009) as it indicates leptokurtosis ( $Ku = 2.21$ ). Levene's test indicates equal variances for the experimental and control groups,  $F(1,37) = .078, ns$ . Independence is also assumed based on the design of the study. Due to the robust nature of ANCOVA (Field, 2009) and the meeting of other parametric assumptions, the analysis of covariance was completed in spite of the kurtosis concern for the experimental group pretest.

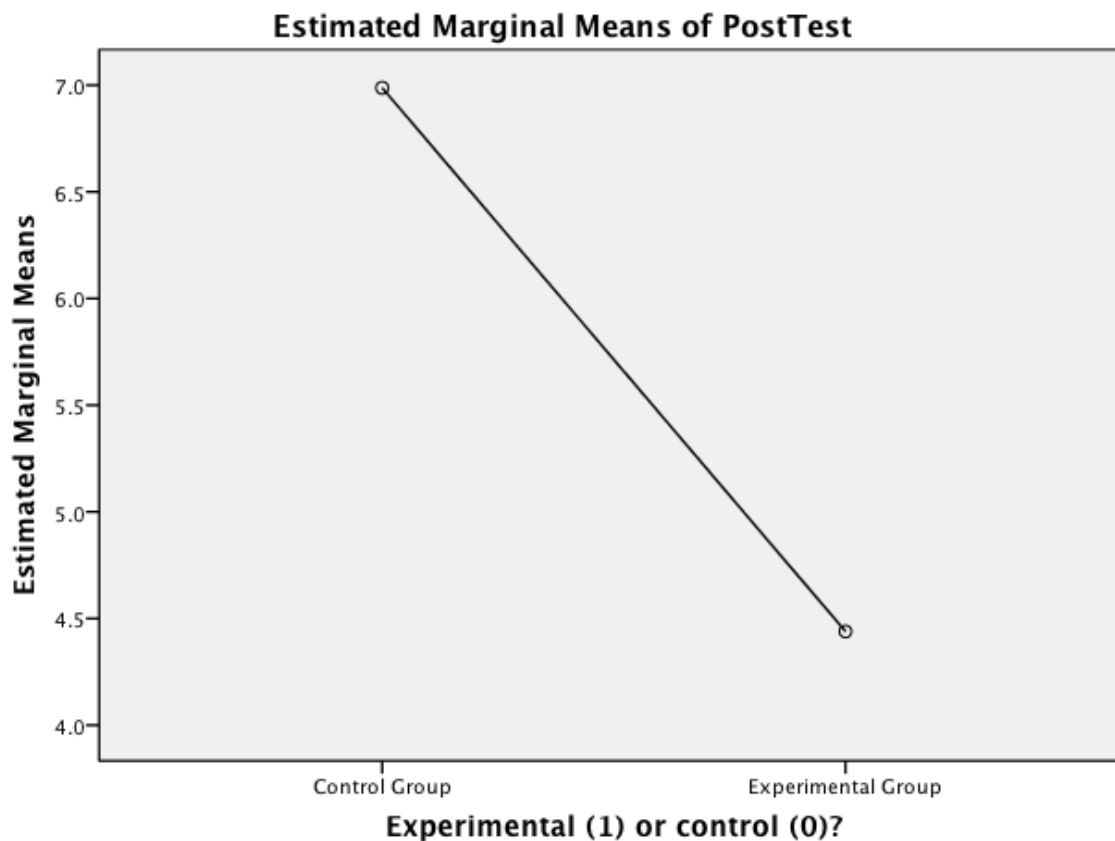
The results of the ANCOVA are displayed in Table 7. The covariate pretest was significantly related to the posttest,  $F(1,36) = 5.81, p < .05, partial \eta^2 = .14$ . There was also a statistically significant difference between the groups after controlling for the effect of the covariate,  $F(1,36) = 7.75, p < .05, partial \eta^2 = .18$ , although as shown in Table 6, the control group actually had the higher mean.

Table 7

*Analysis of Covariance*

Source	<i>df</i>	Mean Square	<i>F</i>	Sig.	Partial Eta Squared
Corrected Model	2	76.03	10.98	.000	.38
Pretest	1	40.22	5.81	.021	.14
Group	1	53.63	7.75	.009	.18
Error	36	6.92			

**Research question 1.** The first research question was “Does metacognitive writing increase the level of mathematical understanding for Advanced Placement (AP®) Calculus students as shown through measurements of achievement?” While a statistically significant difference between the experimental and control groups was found in the analysis of covariance,  $F(1,36) = 7.75, p < .05, partial \eta^2 = .18$ , the estimated marginal mean of the control group was higher than that of the experimental group for the posttest (see Figure 1). Based on the results displayed in Tables 5 and 6, the mean for the control group remained consistent from pretest ( $N = 18; M = 7.44; SD = 2.57$ ) to posttest ( $N = 18; M = 7.44; SD = 3.24$ ). For the experimental group, the mean decreased from pretest ( $N = 21; M = 5.10; SD = 3.13$ ) to posttest ( $N = 21; M = 4.05; SD = 2.36$ ). The null hypothesis was that metacognitive writing has no impact on the mathematical understanding of AP Calculus students. While the null hypothesis was rejected, a statistically significant difference was found between the experimental and control group in the direction opposite of the hoped for result.



Covariates appearing in the model are evaluated at the following values: PreTest = 6.18

*Figure 1.* Estimated marginal means of the posttest indicating a higher mean for the control group than that for the experimental group.

**Research question 2.** The second research question was “What themes emerge from the reflective journal entries of the AP Calculus students?” Of the 21 reflective writing journals completed by students in the experimental group, 11 were analyzed for text themes and sentiment utilizing Semantria® text analysis software. Sentiment scores are determined by analysis of the content of the specific text (Lexalytics, 2015a), labeling the score by its polarity of neutral, negative, or positive. Sentiment analysis evaluates the context of the extracted theme to determine the positive or negative nature of the writing.

Text themes based on calculus concepts addressed over the course of the study were pre-populated into the analysis software by the researcher. A summary of information regarding the number of themes that emerged as well as the sentiment scores is displayed in Table 8. Theme totals ranged from 15 to 26 with 22 as the average number of extracted themes for the 11 participants.

Table 8

*Summary of Theme Extraction and Sentiment*

	Minimum	Maximum	Mean
Number of Themes	15	26	22
Total Sentiment Score	-1.90	.91	*

\*Semantria does not recommend using a simple average when analyzing sentiment across a selection of documents (Lexalytics, 2015b).

In Table 9, some common themes are indicated with the total number of times that particular theme appeared in the writing of the 11 students. Also included in Table 9 is the number of times the theme appeared with a certain sentiment polarity. Sentiment analysis indicated that most students wrote in a generally neutral context. One student's sentiment scores did show indications of more polarized writing. Of that student's 23 extracted themes, 12 were neutral, seven were negative, and four were positive. The remaining students ranged from zero to four negative or positive scores. Since specific themes were pre-populated into the Semantria software, the resulting outputs were as anticipated with common themes matching the main topics covered during instruction in the AP Calculus classes.

Table 9

*Common Themes and Sentiment Polarity*

Theme	Total Theme Count	Neutral Sentiment	Negative Sentiment	Positive Sentiment
Washer Method	16	16	0	0
Limit	16	16	0	0
Integral	13	11	2	0
Trapezoid Rule	6	5	1	0
Shell Method	11	10	0	1
Fundamental Theorem	6	6	0	0

In many entries, students restated concepts and formulas from the day's lesson. In others, students began to explore their understandings or misunderstandings of a process or concept.

One student expressed:

The most significant thing I learned today was in the quiz I took and the test corrections. I saw what I was doing wrong in the definite integrals when it had bounds. I saw that  $dx$  always gets replaced and you can find that by solving for it in  $du$ . I was still kinda [*sic*] confused why I got  $du^2$  when I know that shouldn't be...

Another student shared confusion about the limit definition process, utilizing a specific example to illustrate his or her point of misunderstanding, "Question: throughout the process of the limit definition, I still get stuck at a certain point of the process. I don't

understand how to separate the problem from  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} i + \frac{3x^2}{n^2} i$  to  $\frac{3}{n} \sum i + \frac{3x^2}{n^2} \sum i$ ?

Is that how you would separate them?” A third student shared a different point of confusion regarding when to use a particular method, stating, “I don’t understand the Disc and washer method. How did you know what method is use [sic] for the problem that they are asking?”

Students also wrote beyond clarifications, giving themselves hints to remember topics later as well as asking broader questions. One student renamed a specific method in his or her reflection so that he or she would better remember the concept, making a note to remember the washer method as the “donut method.” In some instances students wrote questions that went beyond the scope of the lesson, such as the student who wrote, “Clear: Thin slices of a shape to figure out the volume of the shape. Unclear: How to figure it out if it is an irregular shape. How does this equation work for all shapes?”

Regarding the same topic, a different student wrote:

It makes sense to me that it needs to be “cut” into little circles to find the volume. And how since there will be alot [sic] of parts that will make it mostly go to infinity we have to use the integration. Unclear: What is unclear to me is if it only works for circulars or does it work for any other shape?

One student’s writing occasionally covered topics outside of calculus. In one entry the student expressed frustration over a lack of sleep and forgetting to bring a lunch or lunch money to school that day.

**Research question 3.** The third research question was “What is the correlation between the students’ number of extracted themes from the reflective journal entries, the sentiment value of those themes, and their scores on the posttest?” Performance on the



posttest was significantly correlated with the number of extracted themes in the reflective writing,  $r = .68$ ,  $N = 11$ ,  $p < .05$ . Since the reflective writing occurred prior to the posttest, the conclusion can be made that theme generation is predictive of performance on the posttest, the non-calculator multiple choice portion of the AP Calculus AB examination. Sentiment was not significantly correlated to the posttest score. A summary of the correlation results is displayed in Table 10. The null hypothesis for the third research question was that there is no positive correlation between the number of extracted themes and a students' score on the posttest. The null hypothesis was rejected.

Table 10

*Pearson's r Correlations for Posttest, Themes, and Sentiment*

	Posttest	Themes	Sentiment
Posttest	1	.68*	.06
Themes	.68*	1	-.36
Sentiment	.06	-.36	1

\*Correlation is significant at the  $p < .05$  level (2-tailed).

**Conclusion**

Three research questions guided the design of this study and the analysis of the collected data. It was determined that there was a statistically significant difference between the experimental and control groups with regard to posttest means, but that the control group maintained a higher mean from pretest to posttest while the experimental group showed a decrease in mean. Common themes were extracted using Semantria software from a selection of the experimental group's reflective writing journals. As expected, common themes related to the main concepts being addressed in the AP

Calculus class during the course of the study. Sentiment of the writings was also explored, indicating that most of the students wrote in a neutral context. A predictive correlation was found between the number of extracted themes and the posttest scores, allowing for the acceptance of the hypothesis for the third research question. The key results of the study as well as limitations and suggestions for further research will be addressed in Chapter 5.

## Chapter Five

### Discussion

#### Overview

Piaget (1950) and Dewey (2010) defined thinking as an active process, not merely a passive one. Students in advanced level high school math classes are often tasked with this very active process regarding problems that are conceptually challenging. The purpose of this study was to determine if reflective writing as a metacognitive practice had an impact on student achievement in a high school AP Calculus class. What follows is a discussion of the key findings of the study, possible implications for theory and practice, the limitations of this research, and suggestions for next steps in exploring metacognitive practice in advanced high school mathematics courses.

#### Key Findings and Possible Implications

**Research question 1.** The first research question was “Does metacognitive writing increase the level of mathematical understanding for Advanced Placement (AP®) Calculus students as shown through measurements of achievement?” Students in the experimental group demonstrated a decrease in mean score from pretest to posttest as compared to the control group, which maintained the same mean score from pretest to posttest. The difference between these groups was statistically significant with a small effect size, but in the direction opposite of that expected based on previous studies that were fairly similar in design and purpose (Bond, 2003; Desoete, 2007; Hudesman et al., 2013; Kramarski & Zodan, 2008). Several researchers in previous studies also observed a decrease in scores, but this finding is less common in the published literature. Naglieri and Johnson (2000) found that the performance of some of their participants with

learning disabilities or mild mental impairments actually deteriorated over the course of their study, although their very specific population cannot be compared to that of this study. In a more comparable approach, Lester et al. (1989) reported that for individual students some posttest scores were lower than pretest scores.

Lester et al.(1989) speculated that the new metacognitive techniques introduced to student participants may interfere with previous methods that were already working for the students. This could be one possible explanation for the decrease in mean score for the experimental group seen in this study, especially when one considers the conclusion of Dahl (2004) that students at this level of mathematics already have metacognitive practices that work for them. Another possible explanation comes from Bandura's (1997) assertion that metacognition is tied to motivation. No analysis of student motivation was completed for this study, but most educators acknowledge the existence of the colloquially named "senioritis" (Carpluk, 2010). This study occurred within the last three months of a school year, with the posttest taken after the actual AP Calculus AB Examination. It is possible motivation played a factor with the experimental group scores, but one would likely expect to see a decrease in both the control and experimental groups if this were the case.

Due to theoretical underpinnings and past research as well as the small effect size for the difference between the groups, it would be difficult to conclude that the practice of reflective writing had a negative impact on the students' mathematical achievement. This would need to be explored with future research to determine if Lester et al. (1989) assertion regarding the interference of new metacognition techniques with previously working strategies has merit.

**Research question 2.** The second research question was “What themes emerge from the reflective journal entries of the AP Calculus students?” Eleven reflective writing journals were transcribed by the researcher and analyzed using Semantria® software. The number of themes ranged from 15 to 26 for individual students. Common themes that occurred numerous times throughout the entries involved key topics presented to the class over the course of the study. Students focused most of their writing specifically on calculus concepts through the three given reflective writing prompts (see Appendix B), maintaining a generally neutral tone based on Sentiment scores. Brown (1994) recognized that effective learners tend to have insights into their own abilities, strengths, and weaknesses. Many of the student entries, including the examples presented in Chapter 4, demonstrate the student’s attempt at analyzing his or her strengths and weaknesses through a point of confusion.

One component missing from this particular study is the teacher use of reflective feedback. Hattie (2003) acknowledged teacher feedback as being the greatest source of influence on student achievement based on analysis of over 500,000 studies. As the focus of this study was specifically on the impact on metacognitive writing, the classroom teacher did not read the student journal entries. From a practical application standpoint, these entries could reveal important information about misconceptions, confusions, and even practical concerns that the instructor could address through the provision of consistent feedback. In this way, reflective writing can be used as a formative assessment strategy, allowing students to consider their own learning and giving teachers insight into what re-teaching may need to occur, what outside resources a student might need to be successful in the course, and what feedback is required for student growth. Hattie (2012)

asserted that teachers need to understand what each student is thinking to be able to provide meaningful learning experiences. Reflective writing can give teachers a window into student thought in order to not only provide feedback, but also structure learning activities that support construction of meaning.

The reflective writing in this study was centered on Pintrich's (2002) concept of "self-knowledge" within metacognition, rather than on strategic knowledge. As such, the experimental group did not participate in the additional problem solving the control group did. Rather, the focus was on a more broad approach to reflection regarding overall understanding of what one learned during the course of a class period. The control group also did not participate in any form of writing for this reason, as the study was not designed to explore the strategic knowledge approach to metacognitive writing that could occur as a student works through a calculus problem.

**Research question 3.** The third research question was "What is the correlation between the students' number of extracted themes from the reflective journal entries, the sentiment value of those themes, and their scores on the posttest?" According to the correlation results, posttest score was significantly correlated with the number of extracted themes in the reflective writing of the 11 students. Since the reflective writing occurred prior to the students taking the posttest, this key finding indicates that the number of extracted themes about which a student wrote was predictive of performance on the posttest. Further exploration of this result could lead to use of theme extraction for prediction of student success on the non-calculator multiple choice portion of the AP Calculus AB examination. The correlation result also begins to indicate that the inclusion of specific written themes may have an impact on future achievement. From a practical

standpoint, this adds to the conversation regarding writing across the curriculum and the different ways in which writing strategies can be implemented to support learning. Reflective writing, with a focus on specific themes, could support future academic success in mathematics classes.

### **Limitations**

Twelve variables that affect internal validity and 12 that impact external validity in experimental and quasi-experimental design studies are described by Gall et al. (2007). Perhaps the most significant limitations of this particular study are the small sample size and non-random assignment of participants. The small sample size limits the statistical power needed to detect a desired effect size and impacts the generalizability of the study. Cohen (1992) suggested a sample size of 85 participants to detect a medium effect size ( $r = .3$ ) and 783 participants to detect a small effect size ( $r = .1$ ) with recommended statistical power of .8 at  $\alpha$ -level of .05. The size of the sample for this study indicates that only a large effect size ( $r = .5$ ) could be detected. A smaller sample size is also likely to be less representative of the population to which one might want to generalize results. Non-random assignment of the participants to the experimental and control groups threatens the internal validity of the study with regard to any cause and effect conclusions that could be made (Gall et al., 2007). Confounding variables, such as student absences and external home factors, also raise concern in analyzing the results of the study.

**Internal validity.** The extraneous variables that can have an affect on internal validity are history, maturation, testing, instrumentation, statistical regression, differential selection, experimental mortality, selection-maturation interaction, experimental treatment diffusion, compensatory rivalry by the control group, compensatory

equalization of treatments, and resentful demoralization of the control group (Gall et al., 2007, p. 382). Several of these variables were likely not a concern for this particular study, including demoralization of the control group, compensatory equalization of treatments, experimental treatment diffusion, and selection-maturation interaction. These, as well as compensatory rivalry by the control group, were likely controlled by not revealing to the participants which group is receiving the “intervention” as both classes participated in an exit activity. High school students typically do not share with other classes the specific activities being done in their academic courses so the risk of diffusion was low.

Experimental mortality was a possible concern due to the high mobility rate of the high school. In the case of the experimental group, three students did not complete the entire study. The control group remained intact. Differential selection was avoided by randomly assigning which class was control and which was experimental. However students could be in specific class periods due to other courses in their schedule. Statistical regression was a possible concern since there were students who fell at the extreme ends of the pretest range of scores, although there was not an issue with instrumentation as the measuring instrument should be consistent from pre- to posttest. History and maturation concerns were possible although the study extended for only six weeks. Testing concerns from pre- to posttest may be an issue but were unlikely as most students would not recall specifics from an exam six weeks prior. One additional statistical concern was raised by the leptokurtosis of the pretest of the experimental group. The meeting of other parametric assumptions and the robust nature of ANCOVA led to the decision to proceed in spite of this concern.



**External validity.** Gall et al. (2007) list extraneous variables that can impact external validity, including generalization, personological variable interaction, explicit description of experimental treatment, the Hawthorne effect, novelty and disruption effects, experimenter effect, interaction of history and treatment effects, measurement of the dependent variable, and interaction of time of measurement and treatment effects. Other extraneous variables likely had a negligible impact on external validity for this particular study, those variables being multiple-treatment interference, pretest sensitization, and posttest sensitization.

The small sample size limits generalizability of results. The diverse backgrounds of students mean a variety of background variables could interact with the treatment effects as well. In terms of ecological validity, the researcher will provide an explicit description of the experimental treatment, allowing for replication if desired (see Appendix B). It is possible that the Hawthorne effect had an impact so special attention to that class was limited to instruction of the writing strategies. Novelty effects were avoided by the use of a variety of strategies so that the novelty of one particular writing process does not wear off over time. Experimenter effect was unlikely as there was no direct interaction between researcher and students. It is possible the fairly innovative nature of the intervention may influence the interaction of history and treatment effects although this is difficult to determine without repeating the experiment at a later time. Measurement of the dependent variable in this particular study limits generalizability to the use of AP Calculus non-calculator multiple-choice items. Finally, this study did not account for the interaction of time of measurement and treatment effects. Future studies could use multiple posttests to determine retention of learning.

### **Suggestions for Future Research**

Exploration of metacognition and reflection in mathematics offers a wide range of opportunities for research, especially with regard to high school students in advanced mathematics courses. This study only scratches the surface of possible work. Ideally, this study would be replicated with a larger sample size and with an experimental design to include random assignment. In addition, it could be beneficial to combine reflective writing with think-aloud or verbal strategies similar to those used in Bond (2003) and Rivard and Straw (2000), but at the advanced high school level. As mentioned previously, the teacher did not read the reflective writings of these participants. A future study may want to explore the teacher impact in using reflective feedback as a formative assessment strategy.

Finally, it would be beneficial to investigate the metacognitive processes of AP Calculus students with a more focused approach. For example, students could be asked to write more specifically during a problem-solving process or in the analysis of a conceptual idea such as the Tangent Line Problem, rather than writing in broader terms about their learning during a class period. This more targeted approach could help students write in a more focused manner about all of the aspects of one particular type of problem or concept, as described in Pintrich's (2002) concept of strategic knowledge. Bangert-Drowns et al. (2004) concluded from their meta-analysis that particularly effective writing strategies include those with specific prompts for student reflection regarding learning processes, confusion, and current knowledge. Focused writing of this type may encourage a deeper processing of the topic, and provide the teacher with more specific feedback about student understanding. From a practical standpoint, this could be

implemented by more specific instructions or guiding questions from the teacher about a single concept, rather than the broader approach of asking a student what they learned overall.

### **Overall Conclusions**

Consideration of human thought is certainly not a new concept, as we know from writings as early as the time of Plato (1973). From introspection to metacognition, thinking about our own thinking is an active and persistent process (Dewey, 2010). Grounded in metacognitive theory, empirical research regarding metacognition and learning has expanded over the past few decades. In the field of education, Flavell (1976), Brown (1982), Kluwe (1982), Bandura (1997), Pintrich (2002), and others explored the impacts of metacognition and its various components on the learning process. Researchers, such as those discussed in Chapter 2, studied the effects of metacognitive practice and measurement on a wide range of student populations, from those receiving Student Support Services (Naglieri & Johnson, 2000) to elementary school students (Bond, 2003) to high-achieving high school students (Dahl, 2004) to college students in remedial mathematics courses (Hudesman et al., 2013). The study presented here contributes an additional layer to the ongoing conversation.

While ultimately the results of this particular study do not align with some of the more promising research with regard to positive impacts of reflective writing on mathematics achievement, it does raise questions about the correlation between written themes and student success. It also exposes the need for continued research with regard to reflective practice among advanced high school mathematics students, possible interference of new metacognitive strategies with previously working ones, and the ways

in which such writings could be used for formative assessment purposes. As we continue in a time of increasing expectations for student achievement, through No Child Left Behind (2001), the Race to the Top Assessment Program (U.S. Department of Education, 2009), and the growth of programs such as Advanced Placement, International Baccalaureate, Cambridge International Examinations, and Advancement Via Individual Determination, it is critical to evaluate a variety of approaches to support student learning. This is especially true with regard to mathematics courses.

Ultimately, thought is one of the ways in which we define our humanity, as so strongly expressed in the simple statement, “Cogito ergo sum” (*I think therefore I am*) (Descartes, 2004, p. 18). Our ability to think upon our own learning and to have feelings and knowledge about that process is one of the things that make human learning so fascinating (Brown, 1994). As the learning process continues to be explored, reflective writing is one approach that shows promise and is worth continued research.

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## Appendix A

## District Letter of Permission for Research

February 18, 2015

To Whom It May Concern:

As a representative for \_\_\_\_\_ Public Schools and \_\_\_\_\_ High School, I hereby give Lindsay O'Neal permission to collect data for completion of her dissertation during the current school year. This data collection will occur with Advanced Placement (AP®) Calculus students at \_\_\_\_\_ High School.

Our district is in full support of the use of writing across the curriculum and formative assessment processes, including in our AP courses. The focus of Ms. O'Neal's data collection is on the effect of metacognitive writing on mathematics achievement. This metacognitive writing component is a part of the regular curriculum of the course. Ms. O'Neal's research will inform not only her dissertation, but also movement forward in support of metacognitive writing practice within \_\_\_\_\_ Public Schools.

Please let me know if you have any questions or need additional information.

Sincerely,

Executive Director for Teaching and Learning



## Appendix B

### Reflective Writing Explanation and Description for Teacher and Students

**What is Reflection?** Reflection is thinking about your learning and learning experiences. It is *not* summarizing main ideas, but it *is* about reflecting on what ideas make sense and what ideas need more clarification.

**What am I doing for Reflection?** During the last five minutes of class every day, you will respond to one of three reflection prompts in your Reflection Journal. Please write with as much detail and thought as possible.

#### **What are the Reflection Prompts?**

***I Learned Statement.*** (Ellis, 2001; Simon, Howe, & Kirschenbaum, 1972).  
When you write an “I Learned” statement, you will begin with “I learned...” and write complete sentences about what you learned in class that day. Be specific and give details!

***Clear and Unclear Windows.*** (Ellis, 2001).  
Draw a vertical line down the middle of the page in your notebook and write “Clear” on the left side and “Unclear” on the right side. On the Clear side, write about what is clear to you about what was covered in class that day—what makes sense. On the Unclear side, write about what is unclear to you about what was covered in class that day—what things you might still be confused about.

***The Minute Paper.*** (Wilson, 1986)  
When you write a “Minute Paper,” you will respond to these two questions:  
1. What is the most significant thing you learned today?  
2. What question is uppermost in your mind at the end of this class session?