Spring 4-14-1983

The Role of Physics in the Christian Liberal Arts University

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The 1983 Winifred E. Weter Faculty Award Lecture

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"Replies to the Voice in the Whirlwind: The Role of Physics in the Christian Liberal Arts University"

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April 14, 1983

This lecture is dedicated to all who have
helped me explore the splendor of our
space and time, and especially
to Evelyn, runner of the
last mile.

1983 WETER FACULTY AWARD LECTURE

Dr. James H. Crichton

Tonight we celebrate the liberal arts at our Christian university through reflection upon the creativity of theoretical physics--the physics of the imagination, the physics of our present time that, by considering worlds unattainable except through mathematical fancy, has led to such deep understanding of our real world. But this great achievement of our era will be put in the context of an ancient and grander wisdom.

Lest we immediately ensnare ourselves in the meshes of mathematics, let us begin tonight's physics lecture by telling two stories from the rich history of our subject. Let us first go back to a clear September evening in Padua, Republic of Venice, 1609. Galileo is peering at the heavens through his newly-built telescope. Although he designed it for commercial and military purposes, he found that it was the night sky that really proved the worth of this new instrument. He saw things never seen before—the rings of Saturn, the moons of Jupiter, and the roughness of the surface of earth's moon. At once, he had the observational data to confirm the heliocentric solar system of Copernicus and to render worthless that curious Aristotelian-Ptolemaic-Thomistic geocentric view. The centrality of the earth and the perfection of the heavens could no longer be maintained--though, of course, the clerical establishment, comfortable with this view, temporarily held the upper hand against Galileo. But the delay was brief and the achievements of observational science triumphed. It was as if he had added another dimension to the realm of human investigation. Bronowski said of Galileo:

> "He asserted that the laws here on earth reach out into the universe and burst right through the crystal spheres. The forces in the sky are of the same kind as those on earth."[1]

A second story, almost certainly fictional, relates events occurring in an orchard in Woolsthorpe, Lincolnshire, England, in the autumn of 1665. The young master of the estate was at home because Cambridge University was closed due to an outbreak of the plague. He was seated under a tree this particular afternoon while working out his ideas in mathematics and physics, trying to extend the work of Galileo. Lost in thought, gazing in the general direction of the moon--maybe he was thinking about the moon's motion about the earth—when his attention was interrupted by a falling apple. As the apple accelerated to the ground, the thought accelerated in Newton's mind that a force—gravity—pulled both apple and moon. He quickly calculated the ratio of the forces (proportional to the acceleration) and found that the gravitational force depended inversely on the square of the distance from the center of the earth. Then, the same law, when applied to the gravitational pull of the sun, accounted for the motions of the planets. Well, it may not have happened in a single afternoon in the orchard, but sometime during that two-year respite, Newton did work out his laws, experimentally verifiable on earth, that applied to the heavenly bodies.

So Galileo and Newton and those that followed them established the laws based on their earthborn observations that enabled representatives of humanity to walk on the rough surface of the moon and to send artifacts to the outermost reaches of the solar system.

Now, in the light of these stories from the history of physics, consider a verse of ancient literature:

> "Do you know the ordinances of the heavens? Can you establish their rule on the earth?"[2]

What is the source of this remarkable question that leaps across the centuries to be answered in our scientific era? The question, from the Book of Job, is addressed to Job by Jahweh speaking out of the whirlwind. Job, the universal symbol of suffering humanity, struggling to understand the situation of loss in which he found himself, is asked by the almighty creator of the universe a question about physics. Clearly, we need to study the context a bit more. Job, in the preceding chapters, had been receiving reasons for his misfortunes from his alleged friends. With all that good worldly counsel, Job might have tried to make some sense of his situation. But then Jahweh speaks, confronting Job with question after question, challenge upon challenge to make sense of the world of nature--the awesome creation--of which Job is such an insignificant part.

> "Where were you when I laid the foundation of the earth . . . Who determined its measurements . . . Have you commanded the morning . . . Where is the dwelling-place of light . . . "

and on and on, covering not only physics, but astronomy, geology, hydrology, meteorology and biology as well.

Some recent interpreters of Job have seen in this discourse a healthy antidote to the "subdue the earth" message of Genesis.[3] They suggest that the cosmos was created for the enjoyment of God and not for subservience to man.

There is so much of creation that Job does not know about simply because it does not concern him; it is for God's pleasure. The earth is the Lord's and the exploiters and polluters should be frequently reminded.

One of the intriguing elements of the extended discourse is the mention of the great beasts, Behemoth and Leviathan. The environmental interpretation would claim that these are typical beings in that part of creation which is for Jahweh's pleasure and not for man's. Standard interpretations suggest that these beasts represent the evil and chaos which trouble mankind, but which ultimately are under control of the Creator. Northrop Frye, a Canadian literary critic, has recently given a striking twist to this suggestion:

> "Cosmologically, the leviathan is the element of chaos within creation: that is, it is creation as we see it now, the world of time and space that extends away from us indefinitely, the limitless expanse that is the most secure and impregnable of all prisons."[4]

We are not here to become mired in competing interpretations of the Book of Job, but one paraphrase that seems to me to put these questions into perspective is: Look, Job, you don't even understand everyday occurrences of the natural world around you: how can you hope to understand as complex a thing as human suffering? But whatever of a host of competing interpretations we choose, these questions remain today as challenges of human power and understanding.

Let us tonight consider the enterprise and achievements of theoretical physics in the light of these challenges from this masterpiece of world (and sacred) literature. The major challenge concerns humanity's finitude, powerlessness to affect the created order. Theoreticians have no wizard's wands or incantations to change the laws of the universe. Push the theorists out of trees and they will fall just as the apples do. But to understand the laws of gravity and motion--that is an achievement that the author of Job had not foreseen. And our understanding comes about because, in our own modest, limited, restricted, confined way, we can engage in acts of creation. No mighty fiats, no "commanding of the morning", but what we can do is to create different worlds with mathematics and imagination.

The constructions in our minds of alternate worlds may be purely for recreation, but we often feel the need to relate them to our real world, to help us better comprehend and control its compelling presence. Popper has called scientific theories "the net which we cast to catch the world" and, as chemist/

philosopher Nash has commented, we endeavor to make the mesh ever finer.[5] Popper's metaphor reminded one writer of some lines written by Sophocles in that age when scientific awareness first dawned:

> "Numberless are the world's wonders, but none more wonderful than man . . . the lightboned birds and beasts that cling to cover, the lithe fish lighting their reaches of dim water, all are taken, tamed in the net of his mind . . . O clear intelligence, force beyond measure!"[6]

So Newton fashioned a net and captured apple and moon and the entire solar system and we continue to weave our nets today. Let us look then at some of the ways we try to capture our world with these nets of theory, creativity and imagination; ways in which we overcome our limitations as finite creatures to answer the voice in the whirlwind as best we can.

First, we realize that it is a wonderful part of creation that we can consider things as disconnected; we can treat different parts or aspects of the world separately. Newton did not have to worry about the forces holding atoms together when he figured the forces that governed the solar system. Those who work with the structure of atoms can neglect gravity. The experimenter in the laboratory can generally control the essential conditions of interest in a limited physical system while ignoring the rest of the universe. It is as if the experimenter is able to say "let there be no gravity" or "let the electromagnetic forces be turned off" or "let the rest of the universe vanish." Magical powers are not needed to investigate the world. (Indeed, if the world were subject to change arbitrarily because of certain magical powers that some people possessed, it would be these powers that would be the subject of investigation.) That the world exists in this separable way allows scientific investigation to proceed and to be productive: Nash has termed this phenomenon the "principle of dissolubility," one of several metaphysical principles which underly the scientific enterprise.[7]

As helpful as it is, that the universe can be investigated a piece at a time, consider what outrageous things theoretical physicists do. When the theoretical physicists attempt to reconstruct reality mathematically, they often encounter infinite quantities in their calculations. Now, "infinity" is certainly one of the great abstractions of the human mind: there is nothing like it in nature. Soviet mathematician Yu. I. Manin has said that "Infinity is not a phenomenon--it is only a word which enables us somehow to learn truths about finite things."[8]

Of course, we should put in here that astrophysicists claim that infinities occur at certain "singularities" where the space-time fabric of the universe is rent asunder—as at the center of black holes or at the Beginning, the Big Bang.[9] With these possible exceptions, we note that the universe is only finite in extent (at least to the extent of our observations), it has lasted for a finite time, and contains a finite amount of matter. Experimenters could never measure an infinite quantity; yet theoreticians deal with infinite masses, electric charges and energies. Fortunately, in the good theories at least, there are ways to handle and remove the infinite quantities, so that the calculated quantities to be compared with experiment are always finite.

Another outrage perpetrated by the theoretical physicists that goes beyond the created order is the use of complex numbers. These mathematical oddities that involve the square root of negative one are necessary for the quantum mechanical analysis of the atomic and subatomic realm. While of course the real world yields measurements which are expressed in terms of real numbers, the theorists have shown that sometimes these measurements are actually real and imaginary parts of complex numbers.

For another example, I will have to give some background in subatomic physics. The nucleus of an atom is composed of particles, protons and neutrons, that attract each other with strong but short-ranged forces. Try to make a nucleus too big and the electrostatic repulsion of the positively-charged protons dominates over the strong forces and renders the nucleus unstable. Thus in our real world, there is a limit to how big atomic nuclei may be, with several important consequences. One is that there are only a hundred or so chemical elements. Another is that the larger nuclei are unstable against such significant reactions as fission, the exploitation of which has unleashed rather awesome powers of destruction.

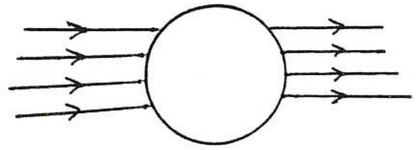
Now imagine turning off the electrostatic force: what would happen? Nuclei could become indefinitely large. Such a system of an unlimited number of protons and neutrons has been dubbed "nuclear matter."[10] Imagine the entire universe filled with matter one hundred thousand billion times as dense as water: that is nuclear matter. Its important properties can be deduced from our information about the strong forces among the nuclear constituents. But why study the properties of this esoteric material? The answer is that the energies that bind real finite nuclei arise from a number of contributions including effects of strong and electromagnetic interactions, surface energies and so on. By constructing the hypothetical nuclear matter, we are in effect turning off all these other contributions and focusing our attention on the strong interactions. We are able to build conceptually the real-world atomic nuclei one effect or contribution at a time, to achieve a deeper understanding of what a real nucleus is.

Now for another outrage: think of what a tiny thing an atom is. Line them up and you get a billion to the inch. Conversely, one of the limitations we have in achieving a complete understanding of the universe is our tremendous size. Our bodies are comprised of billions of billions of billions of atoms. How can we ever hope to learn about such small entities? We cannot take a "cosmic elevator" (suggested by Feinberg and Shapiro[11]) to get down to their scale—we have to base our understanding on the experiences and intuitions developed on our human scale. We know that most of the volume of an atom is occupied by the electrons, whose influences and exchanges constitute chemical binding and reactions. How are we to picture these elusive ubiquitous things that mediate all the energy transactions of living systems, and what's more, energize our television screens?

In some experiments, the electrons show their corpuscular nature—they are particles, as are stones launched from slings, or cannonballs, or billiard balls. But if they are always and only particles, atoms would collapse as the electrons would radiate their energy away. So, in other ways, the electrons behave as waves, as much as waves on a wind-blown lake. (A brief digression on the etymology of "wave": Its similarity to "weave" stems from the Sanskrit word for spider—an interesting comparison of the geometrical forms of a two-dimensional wave pattern and the web of the spider and also a reminder of our "net" metaphor.) But if electrons were only and always waves, they could not be maneuvered to hit television screens in the precise ways that they do.

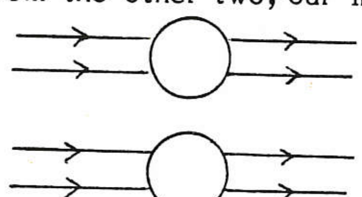
So here we see again the paradox of power and understanding addressed in Job: the theoretician is free to speculate on the consequences of electrons as if they were particles only or waves only, but real electrons behave as both, in a way which defies experience and intuition. What is done about the situation is that a mathematical way of treating wave/particles has been developed. The abstract mathematics of quantum mechanics handles the wave/particle dilemma perfectly.

If we cannot get down to the atomic scale or to the scale of fundamental wave/particle/entities that comprise the atoms, what can we do? We can modify or extend our language by going to abstract mathematics. Let me illustrate one approach (literally) as follows:

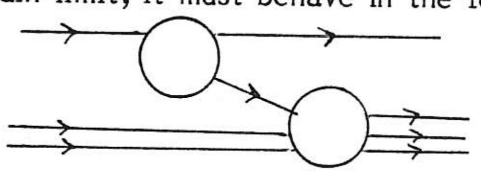


(It is not a spider weaving the web of quantum mechanics!) Each line represents a wave/particle—we could be talking about electrons, protons, neutrons or something more unusual. The circle represents the unknown interaction among the particles. It is the physicists' task to determine what equations, calculations and numerical predictions should take the place of this circle. These mathematical expressions, so represented, are termed S-matrix elements: S for scattering, as the particles interact with each other; matrix element being the technical mathematical term for an array of numbers. Our acquiring of an understanding of the microworld consists of deducing these quantities. For electrons and electromagetic interactions, the theory and experiments are wonderfully complete and coherent. For protons, neutrons and subnuclear constituents and their strong interactions, the amount of experimental data overwhelms the theory, but great strides are being made.

Let us consider some beginning steps in finding out what these S-matrix elements should be like for the strong interactions. A first limitation to be considered is the connectedness, or cluster, structure.[12] When two of the particles are far removed from the other two, our interaction should be depicted by

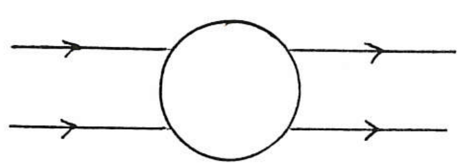


Another constraint on this four-particle-in, four-particle-out S-matrix element is that in a certain limit, it must behave in the following way:[13]



It must factor into a product of S-matrix elements for two- and three-particle interactions, with an appropriate expression for the particle that leaves the first interaction before participating in the second one. Note how in these conditions/limitations/constraints, we are using the principle of dissolubility--we are trying to take the more complicated interactions apart and consider simple fundamental interactions.

Now we see that the most fundamental interaction must be that in which only two particles are involved, diagrammed by



This type of interaction corresponds to the usual experimental situation in elementary particle physics of accelerating one particle and smashing it into a target particle. Remember, it is because the particles are so small that our experience and intuition cannot be brought to bear on direct observation of the interactions, so we must rely on mathematical abstraction to interpret the data. What is observed is the pattern of particles emerging from the region of the interactions--we measure how many particles are scattered through a given angle. This data--the pattern of scattering as a function of angle—is then compared with the theoretical prediction. The theory is then fine-tuned to fit the data.

There is an important set of numbers that mediate the experimental/ theoretical transactions. These are called "phase shifts." They quantitatively represent the effects of the interaction on the wave nature of these "particles." On the one hand, they can be extracted from the data; on the other hand, they can be calculated from the theory. It is thus an important task to obtain these quantities, the phase shifts. For later reference, it should be mentioned here that there are limitations to this analysis. S-matrix elements are complex numbers (they necessarily involve the square root of negative one); physical measurements, of course, involve only real numbers. It turns out that alternate sets of phase shifts may yield the same measurements, but would correspond to different S-matrix elements.[14] Thus an ambiguity arises in which the experimental data would not be able to resolve conflicting theoretical predictions. The existence of such alternate sets of phase shifts constitute the "phase-shift ambiguity" of which more will be said later.

So far then, we have given some examples of ways theoretical physics stands in relation to a world it did not create, yet achieves understanding through creations of the mind, alternative universes developed through imagination and mathematics. Let us spend some more time in another example, the main topic of tonight's lecture, dimensionality.

This physical space--the scene of our scientific investigations, arena of our human activity--is three-dimensional. No fact of our existence is easier to observe or harder to change. We can map out our space north-south, then east-west, then up-down, but there are no further directions to specify. Or, consider a point which traces out a straight line, then the straight line being moved in a transverse direction and tracing out a plane. Take the plane and move it up or down and a solid figure is generated. Beyond that, there are no dimensions in which to continue our geometrical construction. There are three dimensions and no more. We have no experience or intuition for any other dimensionality. This fact is so pervasive and taken for granted that it was unnecessary to give Job a question like: "Have you determined the dimensionality of space? You can count to three, you know that there are three dimensions. Yea, you can even count to four; tell me, can you make a fourth dimension?"

Dimensionality does not have just geometrical or physical implications; consider the growth of human awareness in a third dimension. We humans used to be two-dimensional creatures, in the limited sense that we did not appropriate the vertical dimension into the realm of human activity. Geographically, we were confined to the two-dimensional surface of the earth, staying pretty much close to and above sea level. The vertical dimension seemed reserved for the sacred or mythological. The Tower of Babel may have been an intrusion into this forbidden dimension. The giving of the Law, the temptation of Christ and the Transfiguration occurred at lofty elevations. Ancient Greek metaphysical systems regarded the heavens as perfect and unchanging. No wonder the medieval world was three-layered with a two-dimensional earth as the middle of the sandwich. Dante, on his way to Paradise, climbed up the Mount of Purgatory. Only in the last two centuries have people accepted the challenge of real mountains and left the Terra Firma in balloons and airplanes.[15] The revolution in thought to which Galileo and Newton made major contributions literally opened up the third dimension to human activity. We have searched up and down this dimension and have not found anything sacred or any mythological beings.

In a similar sense, we have added time as a fourth dimension. In ancient times, people used to think in terms of cyclic processes, as in daily and yearly renewals of nature, and not in terms of a linear progression of time. By contrast, the Judeo-Christian tradition, bearing a story of God's activity in history, suggests a linear unidirectional time.[16] Also the ancients had no notion of the immense age of the universe. Following in the tradition of Galileo and Newton, geologists, geophysicists, and astrophysicists were able to push back the start of the time dimension to a singular event some fifteen billion years ago—the Big Bang.[17] Of course to speak of time as a dimension requires a course in Einstein's special theory of relativity. It was Minkowski who recognized the similarity of time to the spatial dimensions and proclaimed: "Henceforth space by itself and time by itself are doomed to fade away into mere shadows."[18]

Minkowski's intuitive interpretation of the special theory was well-confirmed a few years later with Einstein's general theory of relativity. In this towering intellectual triumph, gravity, due to the presence of matter, manifests itself as curved space-time. The effect on our three-dimensional space is sometimes described by imbedding it in a space of higher dimensionality--in the same way the surface of a sphere is two-dimensional but we understand it as a curved surface in three-dimensional space.

Our theoretical physicists of course can construct mathematically worlds of any number of dimensions. Several current lines of research that exploit dimensionality will be summarized later. Let us first delve into a bit of the history of ideas on dimensionality.

Aristotle, in the oldest surviving mention of the dimensions, put it like this: "the line has magnitude in one way, the plane in two ways and the solid in three ways and beyond these there is no other magnitude because the three are all."[19] Algebraists of the 16th and 17th centuries encountered extra dimensions as they sought roots of equations higher than the third order. The German Stifel speaks of "going beyond the cube just as if there were more than three dimensions, which is against nature." John Wallis, English mathematician who gave us the symbol for infinity, declared the extra dimensions to be "monsters in nature, less possible than a chimera or centaur."[20]

But once Descartes had developed analytic geometry, the way was clear to go beyond coordinate systems of three-dimensional space. The power to create new dimensions was a simple matter of counting--let x_1, x_2, x_3 be the coordinates of a point in the first three dimensions--let there be x_4, x_5, x_6 ... x_d to complete the specification of a point in d-dimensional space. Mathematical physicists used this scheme to first describe the motion of bodies in six-dimensional space: three dimensions of spatial coordinates and three dimensions for the components of velocity. The next step was to represent the state of motion of N particles (where N could be any incredibly large number) by a point in a 6N-dimensional space. It was Lagrange, the great mathematician who extended Newton's mechanics, who made this advance.[21] Workers in kinetic theory and statistical physics used these multi-dimensional spaces and worked out volumes and surface areas for 6N-dimensional spheres.

One of the most fruitful applications of extending the dimensionality of space is in quantum mechanics. Mathematicians, mainly Banach and Hilbert, developed infinite-dimensional linear vector spaces just in time for them to be appropriated by the "new" quantum mechanics of the 1920's, that of Heisenberg, Schrödinger and Dirac. Either the wave functions themselves, or the more abstract "state vectors" of the physical system, were usefully described as elements of these infinite-dimensional spaces. (Of course, the systems they represented had their wave/particle existence in three-dimensional physical space.)

Geometers were not to be left out and developed theorems corresponding to those of Euclid for two dimensions.[22] But dimensionality was so much fun that it attracted the attention of others besides physicists and mathematicians. In the 1890's, an English clergyman, E. A. Abbott, wrote an enduring story, Flatland, relating the difficulties creatures in a two-dimensional world would have in comprehending the third dimension that we all enjoy.[23] Others in the era were trying to understand the fourth dimension and how it could be explained to creatures whose experience and intuition were limited to three dimensions. Articles on the fourth dimension were found not only in the mathematics journals at the turn of the century, but also in Harper's, Popular Science and Scientific American. The story with which Plato opened Book VII of the Republic was frequently cited--how the prisoners in the cave mistook their two-dimensional shadows for their real existence, and by analogy how we might be limiting ourselves if we were content with a three-dimensional expression of our existence. Some of the speculators in the fourth dimension were intrigued by Ephesians 3:18: the breadth and length and the height and the depth of the love of Christ. The occult and mystical experience, the ether and as-yet-not-understood laws of physics were discussed in these articles. In 1909, the Scientific American sponsored a contest for the "best popular explanation of the Fourth Dimension" which drew 245 entries.[24] It is interesting to note that none of the prize-winning essays mentioned time in relation to the spatial dimensions, even though this contest was held the year after Minkowski's proclamation that time was indeed the fourth dimension. But popular attention thereafter focused on relativity and there was little further interest in a fourth spatial dimension, except as it helped in explaining the curvature of space-time.[25]

A most interesting application of the ideas of dimensionality outside of mathematics and physics has been in the theology of Karl Heim.[26] Just after World War II, Heim was concerned with the seeming conflict between the Christian faith and modern science in which the latter always seemed to gain ground at the expense of the former. He wanted to put Christian theology in a bomb-proof stance that would not allow further erosion to the advances of science. First, he analyzed the world as seen by scientific investigation and extended the wave/particle duality to a general principle of opposites which characterize our four-dimensional space-time. Our natural world, he said, occupies a polar space of a finite number of dimensions. But there is also the supernatural--in a suprapolar space that transcends the finite dimensionality and polarity of our space. Just as the world of Flatland is imbedded in our three-dimensional space, so is our four-dimensional polar space-time imbedded in the suprapolar space of God. Modern science is free to investigate the polar space all it wants; God remains unshakable in the suprapolar domain. And, as we three-dimensional beings could make rather miraculous interventions in Flatland, so even more could the transcendent Deity of the suprapolar space have expressions of immanence in our space. Just as the God of the ancients dwelt in a dimension inaccessible to human activity, so Heim placed God beyond all the dimensions we know about.

Heim's ideas were further elucidated by William Pollard, whose training and work in physics made him more appreciative of these insights than many other theologians.[27] In Pollard's words:

> " [seeing] the space-time continuum of the natural world as a contingent, restricted framework, immersed or suspended in a larger framework of higher dimension, is admirably suited to accommodate the Biblical view of reality. The essence of the supernatural in Biblical terms is its immediate and intimate contact with the natural at every point and every moment."

What a grand commentary Heim and Pollard make on Psalm 139!

A more recent example of the impact of dimensionality on theology has been given by Mark Peterson of Amherst College.[28] By a careful reading of the Divine Comedy, Peterson finds that Dante could have had a vision of hyperspheres imbedded in a four-dimensional space as his cosmographic model. Peterson argues that there is a topological way to understand the relationship between the two different sets of concentric spheres--one, Aristotelian, with the earth at the center and proceeding outward through the observable heavenly realms to the Primum Mobile, and a second, the Empyrean, with God at the center and different orders of angels at the various levels. If each set of spheres in fact consists of hyperspheres and the two sets are topologically "glued together", then Dante's travels and visions from the earth to the abode of God by way of the Primum Mobile make much more sense than in a more restricted space. Whether the Empyrean of Dante and Peterson is to be identified with the suprapolar space of Heim and Pollard, we can leave to a conference committee of theologians and topologists.

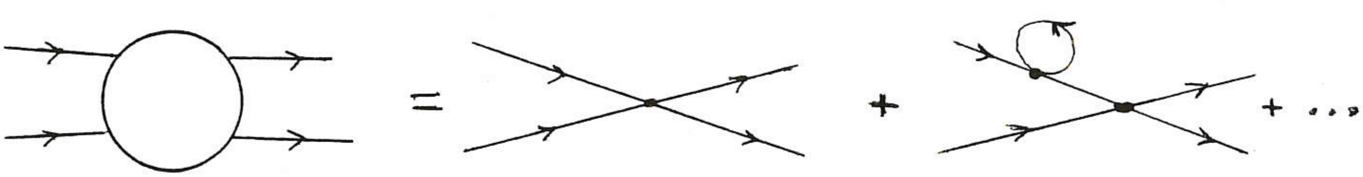
Now it is one thing to talk about the geometry of a d-dimensional space but quite another to investigate the physics that goes on in that space. (Although of course Einstein's theory of general relativity shows how important geometry is to the physics.) Now let us indicate how dimensionality has figured in the physics we have talked about so far. Consider Newton and his inverse square law of gravitation. This inverse square law is more or less directly observable in the behavior of the planets, but it would also follow as the solution of a certain equation--Poisson's equation, which relates gravitational potential energy to the mass that is responsible for that energy. If Poisson's equation is trivially extended to four dimensions, for example, then the gravitational force would vary as the inverse cube of the distance between the attracting bodies.

Or consider nuclear matter: it may well be bound together with different kinds of forces if carried to a space of another dimensionality. One thing certain is that the calculations of the binding energy of nuclear matter would increase in complexity with each added dimension.[29]

Likewise the theoretical work that leads to calculations of S-matrix elements can be dimensionally extended and there turn out to be advantages in doing that, despite the increase in complexity and the apparent inapplicability to the real three-dimensional world which this physics is supposed to explain. About that we'll say more later.

The phase-shift ambiguities, it turns out, would exist in spaces of all dimensionality. This result seemed so exciting to me last year when I worked it out, that I felt that I was a traveler in inter-dimensional space. I had transcended the dimensions! Alas, my further studies of dimensionality led me to believe that, even in three dimensions, the phase-shift ambiguities are rather insignificant.

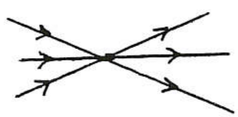
Let us consider a modern use of dimensionality called "dimensional regularization."[30] As we have said, methods to calculate the S-matrix elements usually result in infinite quantities which must somehow be dealt with and thrown out. We say that these quantities are divergent and we must use a renormalization procedure to produce finite results. For example, a contribution to the two-particle-in, two-particle-out S-matrix element might be simply a term which represents the particles interacting at a point. But there are other terms which must be included which are represented by diagrams in which each particle undergoes a self-interaction--emitting and reabsorbing another particle. The factor from each such self-interacting particle line is infinite.



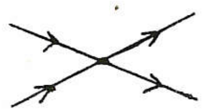
The terms depicted here and a host of others will give infinite, unmeasurable results for the calculations--clearly a bad state of affairs for the theoretical physicists. So what do they do? They go to a universe of a different dimensionality! That is, they calculate the quantities according to rules that would pertain to such a universe. They are able to isolate a specific contribution that is infinite when d equals four, that is, when we apply the calculation to a universe of three-plus-one dimensions. Once these contributions are isolated, they can be cleanly excised and they never show up in the calculational rules. With the new set of rules, the calculations can be done for our three-plus-one dimensional universe with no divergences--the theory has been renormalized.

There is an interesting relationship between renormalizable theories of particle interactions and the dimensionality of space-time. For particles which have no spin, as we have been using in our example previously, the only renormalizable theories are these:

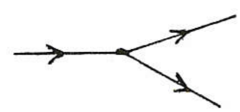
2 + 1 dimensions



3 + 1 dimensions

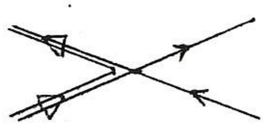


5 + 1 dimensions

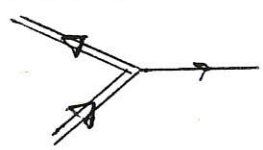


There are no others, in any dimension. If we include particles which have spin, such as electrons (denoted with a double line), we can also have as renormalizable interactions:

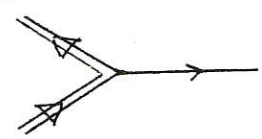
2 + 1 dimensions



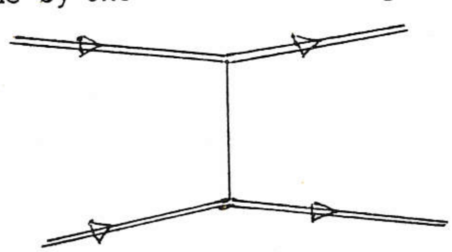
3 + 1 dimensions



1 + 1 dimensions



We have in these diagrams what fundamental interactions we have to deal with in our three-plus-one dimensional world. The electromagnetic interactions, so important in our world, come about by compounding two of these fundamental interactions-- for example by one electron emitting a photon and another electron absorbing it.



We can also see that if we could peer into the quantum field theory books at the Flatland Institute of Theoretical Physics, we would notice that they would be filled with rules for avoiding divergences using the two-plus-one dimensional interactions depicted above. Notice also that in "lineland" there is only one kind of renormalizable interaction and in a world of five spatial dimensions there is not only a single kind of interaction, but no interactions are allowed with particles like electrons. Most interesting is that in a world of four spatial dimensions there would be no renormalizable interactions at all.

Now light consists of photons, which are emitted by charged particles, as in these diagrams. If our charged particles have spin, as do the electron and proton, and our theory of interactions is renormalizable, we can see that our universe must have three or fewer spatial dimensions. So "where is the dwelling-place of light," as Job was asked? It is certainly within our three-dimensional world and not beyond.

Another line of development in theoretical physics is concerned with "unifying" gravitation and other types of interaction such as the strong and electromagnetic.[31] The achievement of a single theory which would incorporate both gravitation and electromagnetism was sought unsuccessfully by Einstein during the last three decades of his life. These "supersymmetry theories" deal with types of particles more complicated than what we have considered so far. To have a theory which has some hope of realizing Einstein's dream is wonderful, but to have to explain away six extra dimensions of space is quite a problem. Einstein himself did not think that going to extra dimensions would be of any help.[32] The present-day theorists, following Kaluza and Klein, "compactify" the unwanted dimensions.[33] It is supposed that they form a space so small as to avoid experimental detection. We can illustrate these subtle, undetectable spaces with a circle on a sidewalk. If you go around and around the circle, you keep passing over the same points. And, if the circle is small enough, you would be stepping on all parts of the circle continuously. These extra dimensions are like circles at every point in our three-dimensional space, yet of such small radius that they have no effect on our atomic and subatomic physics. We would all be existing in these extra dimensions and copies of our three-dimensional selves would be repeated indefinitely along each of them. How fine has the mesh become of the net cast to catch the world! Why do we put up with such subtlety and abstraction? Only because we think that we can have a single theory which accommodates gravitation and all the other interactions, so that our net can catch not only the apple and the moon, but also the electrons and nuclei and all the other wonderful subatomic particles.

As my own modest contribution to dimensional knowledge, let me go into phase-shift ambiguities a bit more. The wave equation which should describe the quantum-mechanical behavior of the scattered particles can easily be extended to more dimensions. Arnold Sommerfeld, the German theoretical physicist who was so instrumental in bringing quantum mechanics to its final form, wrote in an appendix in the last volume of his published Lectures: "After having treated plane and spherical waves in three-dimensional space and plane and cylinder waves in two-dimensional space, we cannot resist the temptation to adapt these formulas to the many-dimensional case."[34] (You can see that even stern, rigorous, hard-working theoretical physicists like to have a bit of fun.) Using Sommerfeld's formulas and the same method as for phase-shift ambiguities in three dimensions,[35] we arrive at the result that these ambiguities occur for space of any dimensionality whatsoever. (For those interested in the details, the relevant equations and a sketch of the derivation is given in an appendix.) It is remarkable that these equations do not even restrict us to an integral number of dimensions. That is bad news because we can guess from our previous examples that to be physically significant, something mathematically interesting should happen as we take our general formulas for arbitrary dimensions to three spatial dimensions or three-plus-one dimensions of space-time. But these multidimensional phase-shift ambiguity formulas have a smooth behavior for all the dimensions, except of course at one dimension, where there is no scattering angle. I think that it is safe to conclude that these ambiguities, then, represent some trivial curiosity in the particular equations we are using and have no deeper meaning.

There are some other ways to relate the laws of physics to the dimensionality of space-time. For example, the British astronomer Arthur Eddington, in the 1930's, attempted to represent the important physical quantities--the size of the universe, the masses of the elementary particles, the charge of the electron, the speed of light, etc.--in terms of pure numbers.[36] He found that there were four of these numbers (one of them I have chosen as my office number--the fine-structure constant, which measures the strength of the electromagnetic interaction) and sought to show that, in an indirect way, there must therefore be four dimensions of space-time. In Eddington's scheme, a different dimensionality would have required a much different set of important physical quantities--a much different physics.

In more recent work, Geoffrey Chew of Berkeley, and others, are trying to relate the elementary particles to certain topological convolutions of the S-matrix elements in three-dimensional space.[37] Starting with certain rules, all of the allowed kinks and twists of space correspond exactly to all the types of particles. This procedure would have been more convincing if it had been carried out before so many of the particles had already been discovered. In its favor is that it does predict some new particles not predicted by other theories. Chew can even calculate an approximate value for the fine-structure constant. If we can believe this topological S-matrix theory, then a different dimensionality of space-time would have different particles and interaction strengths.

What I had wanted to do in this lecture was to present a complete physics of the fourth or higher dimensional universes with the details given in appendices to be submitted as a series of papers to appropriate physics journals. A study has already been made of Flatland physics--or a planiverse--as its author, Alexander Dewdney, a Canadian professor of computer science, calls his two-dimensional universe.[38] Why not do this for higher dimensions--with the appropriate gravitational law (mentioned earlier), Sommerfeld's multi-dimensional wave equations, the phase-shift ambiguities and all kinds of other examples from every field of physics?

It is remarkable how much of our physics does depend on the three-dimensionality of space. It is also remarkable that the concepts and laws and mathematical formulations can be rather easily, if naively, extended to other dimensions. We modify classical mechanics by adding more components to displacement vectors, with corresponding changes in velocities, accelerations, forces and momenta. Newton's laws of motion would be unchanged. For rotational motion, we have to abandon the convenience of the vector- or cross-product, but thereby gain the advantage of seeing more clearly the tensor nature of torque and angular momentum. We can generalize gravity (and electrostatics) by a simple change of Poisson's equation, as mentioned above. Thermodynamics is unchanged except that the concept of volume as an extensive variable would be trivially modified. Of course there would be more translational degrees of freedom contributing to specific heats. In statistical physics, phase spaces would share in the new dimensionality.

In classical electromagnetism, the Maxwell equations would retain their elegance. The tensor nature of the magnetic field would be explicit, while the electric field would have as many components as our new spatial dimensionality. The factor of 4π that shows up in the discussion of rationalized versus unrationalized units would be replaced by some other quantity for the total hypersolid angle in the new space. As Sommerfeld showed, the wave equation can easily be extended for other dimensions, as can the Schrödinger equation of quantum mechanics.

In quantum theory, the commutation relations of position and momentum operators are as simple as in three dimensions; those for angular momentum operators become a bit harder to manipulate and exploit. The complete set of commuting observables would be correspondingly expanded. To treat electrons relativistically, we would have a dimensionally-extended Dirac equation. As to relativistic quantum field theory, we have already mentioned modification of the rules to calculate the S-matrix elements and the relation to renormalizability. (Of course it would be important to work out the Lorentz transformations in our new space-time.)

The summary of changes of the physical laws given above is not intended to be exhaustive, but is merely intended to suggest the feasibility of a program to equip a higher dimensional space with a reasonable physics. Now what kind of universe is it that would have all these physical laws? Consider atoms. Assuming that protons, neutrons and electrons exist, we start with the simplest atom, hyper-hydrogen. Applying the extended Schrödinger equation, we come up with bound states of infinite negative energy, with the electrons collapsed onto the nucleus. Thus, in our four-or-higher dimensional universe, there are no atoms, chemistry or chemists! (The situation reminds us of the crisis faced by physics early in this century when classical physics was unable to account for the stability of three-dimensional atoms. The crisis was averted by the development of quantum theory.)

Suppose, however, that we could devise an equation with more reasonable solutions, so that stable atoms could exist. Then consider the motions of the planets and satellites. One of the questions that was asked soon after Newton's flash of arboreal insight was that if the moon is accelerating toward the earth just as the apple is, why does not the moon fall onto the earth as does the apple? When asked this question in beginning physics classes, we reply that—well, gravity provides just the right centripetal force to accelerate the moon toward the earth, to prevent it from having a straight-line motion which would carry it off into remote space. Then we talk about the moon's angular momentum and how it must remain constant, which means that the moon must continue in orbit. By contrast, what little angular momentum the apple possesses cannot keep it from falling directly to the ground. Think then of the earth's orbit about the sun. Now as the earth approaches the sun it speeds up, but it cannot get closer than a certain miniumum distance because it has too much angular momentum. All these motions of planets and satellites in our solar system are consistent with the laws of motion and the law of gravity stated by Newton.

Now, consider a four-dimensional universe. Let us suppose that an inverse cube law of gravitational force applies as mentioned before. With this gravitational force, it would probably happen that stars would form and maybe even planets. We would want planets for our four-dimensional universe because we need suitable environments for living beings, even creatures who would, in their idle time, dream up a physics that might work in a fictitious three-dimensional universe. The unfortunate thing is that angular-momentum conservation would not keep our hypothetical planet from routinely coming arbitrarily close to our hypothetical sun. The four-dimensional apples would get baked (and vaporized) right on the tree every so often, not a healthy state of affairs. Such a four-dimensional world could certainly not support life as we know it.[39]

Of course, we could postulate an inverse-square-law force for the higher dimensional universe, but we would thereby do violence to Poisson's equation and to the natural extension of Einstein's equations of general relativity, of which Poisson's equation is a first approximation. Or we could try to come up with some exotic form of life, if we wanted our hypothetical universe to be populated. Or, we could give up our fantasizing about these higher dimensions.

These findings, somewhat disappointing but certainly harmless to us three-dimensional beings, opens up some interesting issues in the philosophy of science. For example, the German-American philosopher Hans Reichenbach, in discussing the nature of our knowledge about the spatial dimensions, states:

> "One might try, for instance, to regard the three-dimensionality as a consequence of certain conditions of equilibrium of matter. This condition would be justified if the three-dimensional order of matter could be shown to be the only stable order. Any such proof presupposes certain laws of nature which can be formulated independently of the dimensionality of space. Such a proof might read: If space has n dimensions, and it is a general law of nature that the attraction between masses varies inversely with the (n-1)th power of their distance, then the dimensionality of space must be n = 3, since otherwise the motion of the planets and also the arrangement of the masses of the stars would not be stable."[40]

This idea was picked up by British astrophysicist, R. G. Whitrow. He added another dimension to the argument, as it were, by claiming that fewer dimensions than three would limit the complexity of the nervous systems of living beings and not allow them to observe and reflect upon their universe.[41] Whitrow concludes that "the number of dimensions of physical space is necessarily three, no more and no less, because it is the unique natural concomitant of the evolution of the higher forms of terrestrial life, in particular of Man, the formulator of the problem."

(Whitrow's argument has been attacked by Dewdney, who maintains that the brains of Flatlanders--the inhabitants of the Planiverse--and their computers can be sufficiently complex, but somewhat slower than our three-dimensional versions.)

Whitrow's attempt to relate dimensionality and the existence of human observers was an early version of the so-called "anthropic principle." Proposed by astrophysicist Brandon Carter in 1970, this principle asserts that "what we can expect to observe in the universe must be restricted by the conditions necessary for our presence as observers."[42] Thus a universe with four spatial dimensions could not give rise to living, conscious beings--the observers--but one with three spatial dimensions could. So what we in fact observe is...the three spatial dimensions.

Most of the discussion informed by the anthropic principle has to do with the size and age of the universe, the density of matter and of electromagnetic energy and the strengths of various interactions. It is worth noting that if any of these physical quantities were much different, our universe would not have life as we know it. For example, if the strength of gravity were much weaker, stars would not get hot enough for the fusion reactions that warm us on these fine spring days, nor would they produce, as some must have long ago, those atoms of carbon, nitrogen and oxygen which comprise our DNA and its exquisite elaborations.[43] If gravity were too strong, stellar lifetimes would be too short to permit the leisurely evolution of complex life on dependent planets. So why is gravity the strength it is? So that physicists may sit under trees and think about it. That is the anthropic principle in a nutshell (or an apple peel). Thus it is for gravitation and also for all the interactions and various properties characterizing our universe, including dimensionality. If they were different, there would be no life.

To some, the existence of this fine-tuning of the physical constants that allows life in our universe seems to be the evidence for design--support for the theologians' argument for the existence of God. Thus it appears to Freeman J. Dyson, one of the theoreticians who worked out the renormalization of the theory for electromagnetic interactions--at least, it is evidence for a mind behind the universe.[44]

To many scientists, a more attractive view is in terms of the rather bizarre many-worlds interpretation of quantum mechanics, in which the universe is continually branching into parallel universes representing all possible outcomes of any quantum interaction, ever since the Big Bang.[45] We are tonight in one relatively small subset of these parallel universes, but most of them (an uncountably infinite number of them) would have the wrong values of the important physical quantities and they would be lifeless and observerless. (If we restrict these outrageously abundant parallel universes to the same dimensionality as ours, then all of them may be accommodated together in one super-universe with just one extra dimension. But we have suggested in this lecture the possibility of universes with any number of dimensions, so we would not want to restrict the dimensionality of the super-universe.) To counter the argument from design, one would simply point out that the overwhelming majority of universes are ill-designed for living beings. (Of course, the eye of faith would see these "worthless" universes as precious to the Creator, fulfilling a purpose that man cannot fathom.)

What are we to make of this state of affairs? Are there more dimensions, or not? Do we understand dimensionality well enough to translate our physical laws to universes of other dimensions? In other words, can we mentally create higher dimensional universes? My present feelings about questions like these can be summarized in terms of three rather basic observations (or conclusions):

1. We exist in a three-plus-one dimensional space-time universe.
2. The physics we have discovered fits this universe very well and would not work in universes of other dimensionality.
3. This universe contains living beings who understand and appreciate the physics of this universe and can understand why it would not work in universes of different dimensionality.

As physicist John Wheeler asks, "What good is a universe that is not observed?"[46] Create new laws of physics, create new universes? Hardly--in our efforts to create new worlds in our imagination, we cannot prescribe a physics for them which would include the possibility of living, conscious, thinking beings who would appreciate the physics. We return to our own universe, secure in its laws, awestruck by our ability to discover and understand them; our creaturehood reaffirmed; our own voices mute before the mighty voice of the whirlwind. Our net has been cast out and pulled in and we find in it our whole universe with its physical laws and its three-plus-one dimensions, but nothing more. The same laws and dimensions apply to the apple and the moon and beyond. We have looked into the universe with telescopes more penetrating than Galileo's and found nothing in the heavens that was not on earth or in the minds of those on earth.

Galileo pointed out that the book of nature is written in the language of mathematics. Eugene Wigner, an important contributor to quantum theory, marvelled at "the unreasonable effectiveness of mathematics in the natural sciences."[47] Einstein declared that "the most incomprehensible thing about the world is that it is comprehensible."[48] A contemporary theoretical physicist, Heinz Pagels, proclaims that the quantum theory is a "cosmic code," a message from the universe to serve as a program for social and cultural evolution, if we could only decipher it.[49] British biologist Sir George Porter asserts that "the highest wisdom has but one science, the science of the whole, the science explaining the Creation and man's place in it."[50] A theologian, T. F. Torrance, looks for "a massive new synthesis . . . [to] . . . emerge in which man, humbled and awed by the mysterious intelligibility of the universe which reaches far beyond his powers, will learn to fulfill his destined role as the servant of divine love and the priest of creation."[51]

Who replies to the challenge of the voice in the whirlwind and what are we to say? The challenges and the questions in effect answer themselves. Things are the way they are so that we may be here to listen to that voice. And now we stand ready for new questions and new challenges.

As Job said, "I have spoken once, and I will not answer; twice, but I will proceed no further."[52]

Appendix: Phase-Shift Ambiguities in a Space of Arbitrary Dimensionality

In this appendix, we present the important steps in obtaining the equations for phase-shift ambiguities for spin-independent scattering in a space of d dimensions. First, we obtain an expression for the scattering amplitude in terms of phase shifts. Then, limiting the partial-wave expansion to the first three terms, we exhibit the equations which give the ambiguities. Some numerical examples are calculated and discussed.

We proceed as in three-dimensional scattering.[53] For a potential which tends to zero sufficiently rapidly as $r \longrightarrow \infty$, there is a solution of the Schrödinger equation of the form:

$$\Psi(r,\Theta) \xrightarrow[r \to \infty]{} e^{ikr\cos\Theta} + \frac{f(\Theta)}{r^{(d-1)/2}} e^{ikr} \qquad (1)$$

The first term represents a "plane wave" in the d-dimensional space—a beam of particles of momentum $\hbar k$ traveling in the direction $\Theta = 0$. The second term is interpreted as a beam of particles emitted radially outward, "the scattered wave," the r-dependence determined by particle conservation through a hyperspherical shell whose area is proportional to r^{d-1}.

Using solutions to the wave equation for d-dimensional space,[54] the standard treatment yields the scattering amplitude $f(\Theta)$ in terms of the phase shifts $\{\delta_\ell\}$:

$$f(\Theta) = C_d\, k^{-(d-1)/2} \sum_{\ell=0}^{\infty} (2\ell + d-2)\, e^{i\delta_\ell} \sin\delta_\ell\, P_\ell(\cos\Theta \mid d-2), \qquad (2)$$

with the numerical coefficient:

$$C_d = 2^{(d-3)/2}\, \Gamma((d-2)/2)\, \pi^{-\frac{1}{2}} \exp(-i\,(d-3)\pi/4). \qquad (3)$$

In Eq.(2), the functions P_ℓ are called by Sommerfeld "(d-2)-dimensional zonal spherical harmonics" and are otherwise known as Gegenbauer polynomials. The first few are as follows:

$$P_0(\cos\Theta \mid d-2) = 1, \quad P_1(\cos\Theta \mid d-2) = (d-2)\cos\Theta,$$
$$P_2(\cos\Theta \mid d-2) = \frac{d(d-2)}{2}\cos^2\Theta - \frac{d-2}{2}. \qquad (4)$$

(For d=3, these polynomials are the familiar Legendre polynomials.)

The phase-shift ambiguities are obtained by using only the first three terms in Eq.(2), then expressing the generalized scattering cross-section $|f(\Theta)|^2$ in terms of the three phase shifts.[55] It is found that two different sets of phase shifts can give the same $|f(\Theta)|^2$. Equations to obtain these two different sets, $\{\delta_0, \delta_1, \delta_2\}$ and $\{\delta_0', \delta_1', \delta_2'\}$, exploit the orthogonality of the Gegenbauer polynomials. It is convenient to define

$$\alpha_0 = \delta_0 + \delta_0', \quad \alpha_1 = \delta_1 + \delta_1', \tag{5}$$

$$\varepsilon_0 = \delta_0 - \delta_0', \quad \varepsilon_1 = \delta_1 - \delta_1'$$

It is then straightforward to determine the two sets for a given δ_2 $(=\delta_2')$:

$$\tan \alpha_0 = \sin \delta_2 \cos \delta_2 \, / \, ((d+2)^{-1} - \sin^2 \delta_2) , \tag{6}$$

$$\alpha_1 = \delta_2 + \pi/2, \tag{7}$$

$$\cos \varepsilon_0 = \frac{d-1}{2} \frac{\sin \alpha_1}{\sin(\alpha_0 - \alpha_1)} - \frac{d^2}{2(d-1)} \frac{\sin \alpha_1 \sin(\alpha_0 - \alpha_1)}{\sin^2 \alpha_0} + \frac{1}{2d(d-1)} \frac{\sin(\alpha_0 - \alpha_1)}{\sin \alpha_1}, \tag{8}$$

$$\cos \varepsilon_1 = \frac{d-1}{2d} \frac{\sin \alpha_0}{\sin(\alpha_0 - \alpha_1)} + \frac{d}{2(d-1)} \frac{\sin(\alpha_0 - \alpha_1)}{\sin \alpha_0} - \frac{1}{2d(d-1)} \frac{\sin \alpha_0 \sin(\alpha_0 - \alpha_1)}{\sin^2 \alpha_1} . \tag{9}$$

Ambiguities in the choice of quadrants for ε_0 and ε_1 are resolved by the condition:

$$\sin \alpha_0 \sin \varepsilon_0 + d \sin \alpha_1 \sin \varepsilon_1 = 0. \tag{10}$$

As indicated in the original paper, there is a limited range of values of δ_2 for which the ambiguities occur in three dimensions and there is a similar restriction in other dimensions. Extending the method of Atkinson et al,[56] the limits of $\sin \delta_2$ between which ambiguities occur are roots of the equation:

$$(d-1)^2(d+2)^3x^4 - 2d(d-1)(d+2)^2x^3 + 2(d+2)(d^2-2)x^2 - 2(d-1)(d+2)x + d = 0. \tag{11}$$

As an example, solving Eq.(11) for d=3, 4, and 5, we obtain:

d	δ_2 minimum	δ_2 maximum
3	12.53°	24.15°
4	10.23°	16.08°
5	8.65°	12.22°

For any δ_2 in the ranges given above, phase-shift ambiguities in the appropriate dimension can be obtained by routine application of Eqs. (5)-(10).

It can be shown that Eqs.(5)-(11) also hold for d=2, even though Eqs.(1)-(4) are not appropriate. It is obvious from Eq.(4), for example, that the Gegenbauer polynomials, except for P_0, vanish identically when d=2. The correct expression for the scattering amplitude is:

$$f(\theta) = (1/2\pi ik)^{\frac{1}{2}} \sum_{m=0}^{\infty} \epsilon_m \cos(m\theta) \exp(2i\delta_m - 1) \qquad (12)$$

where $\epsilon_0=1$ and $\epsilon_m=2$ for $m=0$.[57] (The appropriate polynomials are thus those of Chebyshev.) The range of values of δ_2 for which there is an ambiguity in two dimensions is:

$$16.28° \leq \delta_2 \leq 55.59° .$$

It should be noted that Eqs.(5)-(11) can be solved for any real value of d greater than one, i.e., we are not restricted to an integral number of dimensions. That the phase shift ambiguities do not behave in some singular or unique way at physical values for the dimensionality strongly suggests that they are devoid of physical significance. They remain as mathematical curiosities, with some slight nuisance value to those who try to obtain phase shifts from the scattering data.

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