


Spring June 5th, 2017

Discovery Learning Plus Direct Instruction Equals Success: Modifying American Math Education in the Algebra Classroom

Sean P. Ferrill Mr.
Seattle Pacific University

Follow this and additional works at: <https://digitalcommons.spu.edu/honorsprojects>

 Part of the [Algebra Commons](#), [Educational Methods Commons](#), [Science and Mathematics Education Commons](#), and the [Secondary Education Commons](#)

Recommended Citation

Ferrill, Sean P. Mr., "Discovery Learning Plus Direct Instruction Equals Success: Modifying American Math Education in the Algebra Classroom" (2017). *Honors Projects*. 63.
<https://digitalcommons.spu.edu/honorsprojects/63>

This Honors Project is brought to you for free and open access by the University Scholars at Digital Commons @ SPU. It has been accepted for inclusion in Honors Projects by an authorized administrator of Digital Commons @ SPU.

*Discovery Learning Plus Direct Instruction Equals Success: Modifying American Math
Education in the Algebra Classroom*

by

SEAN P. FERRILL

FACULTY ADVISOR, STEVE JOHNSON

SECOND READER, ROBBIN O'LEARY

A project submitted in partial fulfillment
of the requirements of the University Scholars Program

Seattle Pacific University

2017

Approved _____

Date _____

Abstract

In light of both high American failure rates in algebra courses and the significant proportion of innumerate American students, this thesis examines a variety of effective educational methods in mathematics. Constructivism, discovery learning, traditional instruction, and the Japanese primary education system are all analyzed to incorporate effective education techniques. Based on the meta-analysis of each of these methods, a hybrid method has been constructed to adapt in the American Common Core algebra classroom.

Acknowledgements

To Dr. Steve Johnson, thank you for sticking with me as my advisor these four years, for your guidance and comments on my honors undergraduate thesis, and for teaching me in my first class that calculus is just algebra.

To Dr. Robbin O’Leary, thank you for the encouragement along the year of writing this project, for the regular comments and suggestions you provided me without fail each month, and for your willingness to be available to me for help when I needed someone to talk with.

To Dr. Owen Ewald, thank you for encouraging me to remain in the University Scholars program when I felt ready to give up and for providing an outside view on mathematics.

To Taya Schweizer, thank you for being my mentor teacher, introducing me to the work of Dr. Jo Boaler, and teaching me to love mathematics when I was your student.

To my loving parents, thank you for supporting me along the way as I pursued my degree, for listening to me when I would complain about writing this project, and for raising me to never give up, no matter how hard the challenge appeared.

To Pauline Walker, thank you for your constant support and for being a shoulder to lean on when I felt overwhelmed by everything in the process of researching and writing. May we celebrate many years of life together.

To my University Scholars cohort, thank you for being willing ears as I expressed my struggle in writing my thesis and thank you also for the opportunity to celebrate our friendships together while taking a break from the projects with Pizza and Play. I hope I was as good a friend to you.

Introduction

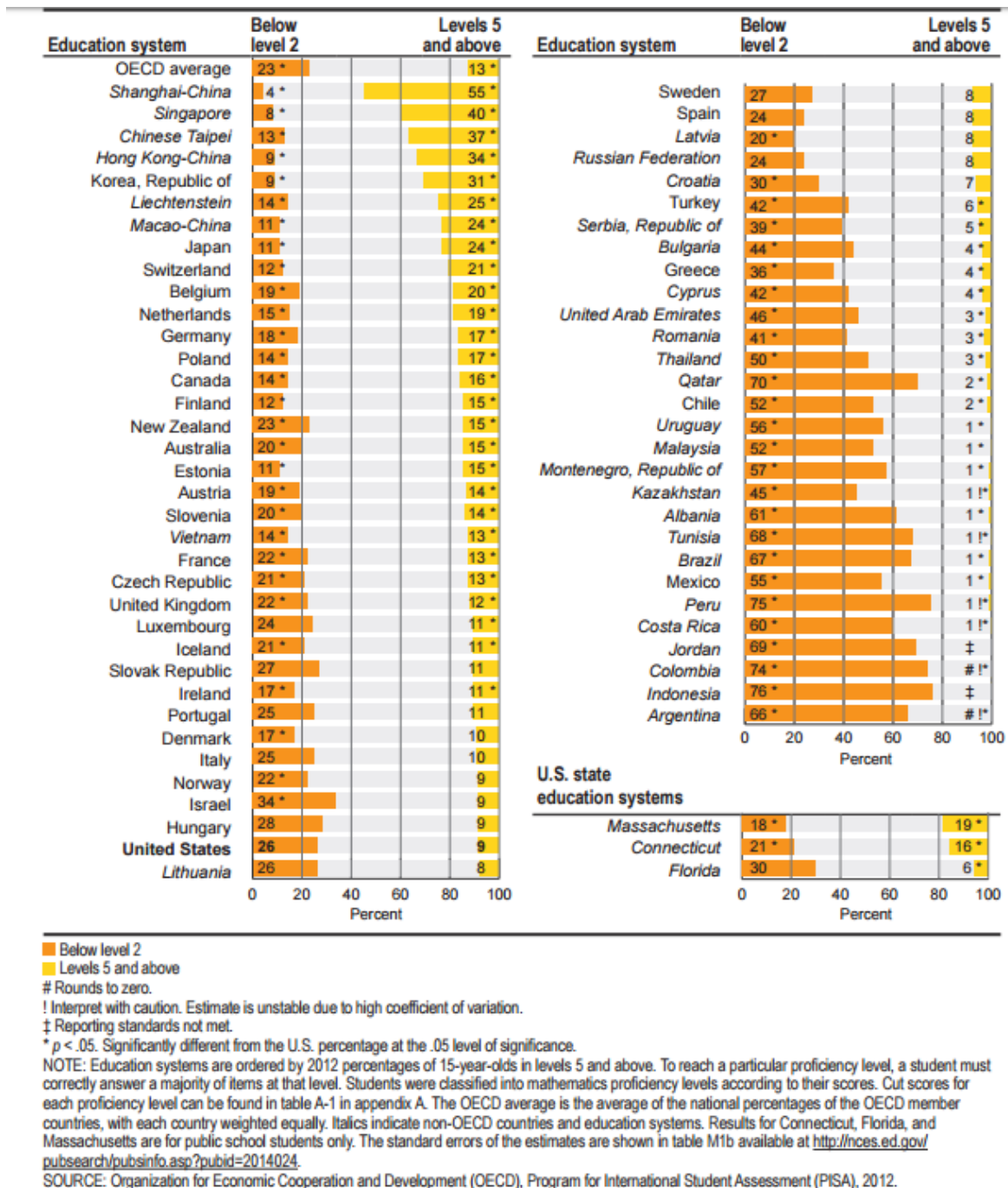
When one considers the goal of education, the three “R’s” are often cited. Reading, writing, and arithmetic rule the educational world. But once students depart their secondary education, we begin to see how they drift away from the final “R.” Mathematics, especially algebra, has been a contentious subject for students throughout the United States. In both high school and college, algebra is the most commonly failed course taken (St. George 2015; Kay 2016, Garnick 2009). Is this the nature of mathematics that it must be so difficult that students are doomed by its very requirement to graduate? The great scientist and author Isaac Asimov (1961) disagreed when he declared “algebra is just a variety of arithmetic” (p. 5). As we consider what it means to be capable of mathematics, we are brought to the question of numeracy—mathematical literacy. Even the definition of numeracy requires additional information. Numeracy to some is simply comfort with arithmetic (Hacker 2015). To others, it is the memorization of formulae and the capability to apply them when prompted. Another definition is “asking questions, drawing pictures and graphs, rephrasing problems, justifying methods, and representing ideas, in addition to calculating with procedures” (Boaler 2015, p. 67). These are still not the end of the definitions of numeracy, as even others declare that numeracy is the capability to think critically when faced with a mathematical context (Mathematics Expert Group 2010, p. 4). Beneath each of these interpretations of numeracy is a desire to understand where high school algebra is included in the definition.

When discussing numeracy, it is important to consider the Organization for Economic Co-operation and Development (OECD) and their tri-annual literacy exam, the Program for International Student Assessment (PISA). PISA examines student content area literacy in member and partner nations of OECD. Of particular note for this study is the data from PISA

2012, which had mathematics as its primary strand. In the PISA 2012 study, numeracy was defined as

An individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged, and reflective citizens (Mathematics Expert Group, 2010, p. 4).

Using the data gathered from PISA 2012, it appears that the United States is struggling badly at maintaining a numerate student population. Students' results from the PISA exam fall into six level categories, numbered one through six and scored 0-1000 (Kelly et al 2013, p. 2-3). Students must score at least a level two to achieve basic numeracy status, while a score of levels five or six indicate the student is vastly ahead in numeracy. Compared with the OECD averages, the United States' students represent a larger sample of the level ones than the average and a significantly smaller proportion of fives and sixes than average. We also see from the data gathered that the United States' average numeracy score is 481, while the lowest cutoff for a level three numeracy is a score of 482 (Kelly et al 2013, p. 3, 9 14). Thus, on average, students in the United States are only at basic numeracy levels, though our average is close enough that we could say we are at an average of level three numeracy. Below, we see two of the graphs the National Center for Education Statistics put together detailing the comparison of the United States and the rest of the surveyed countries.



Graph from National Center for Education Statistics, U.S. Department of Education (Kelly et al 2013, p. 14).

Table 1. Average scores of 15-year-old students on PISA mathematics literacy scale, by education system: 2012

Education system	Average score	Education system	Average score
OECD average	494 ○	Lithuania	479
<i>Shanghai-China</i>	613 ○	Sweden	478
<i>Singapore</i>	573 ○	Hungary	477
<i>Hong Kong-China</i>	561 ○	Croatia	471 ▼
<i>Chinese Taipei</i>	560 ○	Israel	466 ▼
Korea, Republic of	554 ○	Greece	453 ▼
<i>Macao-China</i>	538 ○	<i>Serbia, Republic of</i>	449 ▼
Japan	536 ○	Turkey	448 ▼
<i>Liechtenstein</i>	535 ○	<i>Romania</i>	445 ▼
Switzerland	531 ○	<i>Cyprus</i>	440 ▼
Netherlands	523 ○	<i>Bulgaria</i>	439 ▼
Estonia	521 ○	<i>United Arab Emirates</i>	434 ▼
Finland	519 ○	<i>Kazakhstan</i>	432 ▼
Canada	518 ○	<i>Thailand</i>	427 ▼
Poland	518 ○	Chile	423 ▼
Belgium	515 ○	<i>Malaysia</i>	421 ▼
Germany	514 ○	Mexico	413 ▼
<i>Vietnam</i>	511 ○	<i>Montenegro, Republic of</i>	410 ▼
Austria	506 ○	<i>Uruguay</i>	409 ▼
Australia	504 ○	<i>Costa Rica</i>	407 ▼
Ireland	501 ○	<i>Albania</i>	394 ▼
Slovenia	501 ○	<i>Brazil</i>	391 ▼
Denmark	500 ○	<i>Argentina</i>	388 ▼
New Zealand	500 ○	<i>Tunisia</i>	388 ▼
Czech Republic	499 ○	<i>Jordan</i>	386 ▼
France	495 ○	<i>Colombia</i>	376 ▼
United Kingdom	494 ○	<i>Qatar</i>	376 ▼
Iceland	493 ○	<i>Indonesia</i>	375 ▼
<i>Latvia</i>	491 ○	<i>Peru</i>	368 ▼
Luxembourg	490 ○		
Norway	489		
Portugal	487		
Italy	485	U.S. state	
Spain	484	education systems	
<i>Russian Federation</i>	482	<i>Massachusetts</i>	514 ○
Slovak Republic	482	<i>Connecticut</i>	506 ○
United States	481	<i>Florida</i>	467 ▼

○ Average score is higher than U.S. average score.

▼ Average score is lower than U.S. average score.

NOTE: Education systems are ordered by 2012 average score. The OECD average is the average of the national averages of the OECD member countries, with each country weighted equally. Scores are reported on a scale from 0 to 1,000.

All average scores reported as higher or lower than the U.S. average score are different at the .05 level of statistical significance. Italics indicate non-OECD countries and education systems. Results for Connecticut, Florida, and Massachusetts are for public school students only. The standard errors of the estimates are shown in table M4 available at <http://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2014024>.

SOURCE: Organization for Economic Cooperation and Development (OECD), Program for International Student Assessment (PISA), 2012.

Table from National Center for Education Statistics, U.S. Department of Education (Kelly et al 2013, p. 15).

These two graphics confirm that the United States is struggling in mathematics compared with a sizable portion of the world. Unfortunately, based on prior PISA testing data, it is

apparent that we are making either minimal or no progress in improving our mathematics learning in schools.

Although PISA serves as a valuable insight into the state of mathematics education in the United States, we must remember that PISA is not the final word on American methods. Currently, forty-two states use the Common Core State Standards (CCSS) as their measurement of student success in mathematics. Thus, despite the information provided by PISA, the assessments our schools will use are the test data related to the Common Core. However, PISA data remains useful in measuring American mathematical progress compared with the international community. I have chosen to use the examination as a starting point to examine successful educational programs for the following reasons. PISA provides the same test (translated as needed) to every student. OECD has released sample questions from the specific tests, and ensures the test results are readily accessible. While some people, such as Dr. Andrew Hacker (2016, p. 153-7) consider comparing ourselves to the rest of the world to be detrimental, understanding our position in global mathematics may lead to our more local assessments improving. PISA ought to be used as a guide to develop stronger methods of teaching rather than as a final word on mathematics learning.

In the United States, a culture of innumeracy has grown over the past century. Students who would be ashamed to admit that they could not read or write often brag about their inability to operate with numbers (Paulos 1988). At the same time, there is a push for lowering our standards in response to this aversion to mathematics. Near the forefront of this mathematics-hostile approach is Dr. Andrew Hacker, professor of political science and economics. Prior to publishing his recent book, *The Math Myth and Other STEM Delusions*, Hacker penned an opinion piece where he declared American mathematics ought to abandon so-called “advanced

algebra.” When Hacker (2016) describes what he considers essential mathematics, he claims that arithmetic and mathematics are separate (p. 7). A major component of his argument is how the United States ought to focus on his definition of arithmetic rather than algebra in high school. Thus, students would study how to operate with arithmetic to solve their problems. Hacker’s argument runs into a few problems upon inspection. Under arithmetic, he opts to include statistics. In a cursory examination of any statistics course, to calculate even a normal distribution—let alone a standard deviation—requires a firm grasp of algebra. While one might be able to use arithmetic to perform basic interpretations of data, without algebra students would be incapable of creating their data.

As Hacker turns his attention to the SAT, he subtly conceals his data. He claims that one method to measure top performing students within content areas in the United States is a 700 or more score on the SAT (Hacker 2016, p. 85). For the moment, we will not consider how SAT scores follow a normal distribution, since Hacker chooses not to include this data. Based on the data he received, he found that of the total student population that identifies as either white or black, 5.4% of the students scored at least 700 on the mathematics portion of the SAT. Using that number, he then analyzed how many of those high performing math students scored a 700 or more on the reading section. Of that 5.4% of the total students, 36% of that portion also were high performers on reading. Thus, using Hacker’s provided arithmetic, I found that the total students who scored a 700 or more on both parts was roughly 1.9%. Here, Hacker now manipulates his data. He claims that of the total number of students who scored at least a 700 on the reading section, 44% of them also scored a 700 or more on the mathematics portion. It is obvious, Hacker claims, that reading is a better indicator of mathematics success than mathematics is of reading. However, note that Hacker neglects to list what percent of the total

students actually were his high performers in reading. We know the same pool of students were analyzed in this study. Thus, we know that the 36% and 44% are the same value of the 1.9%.

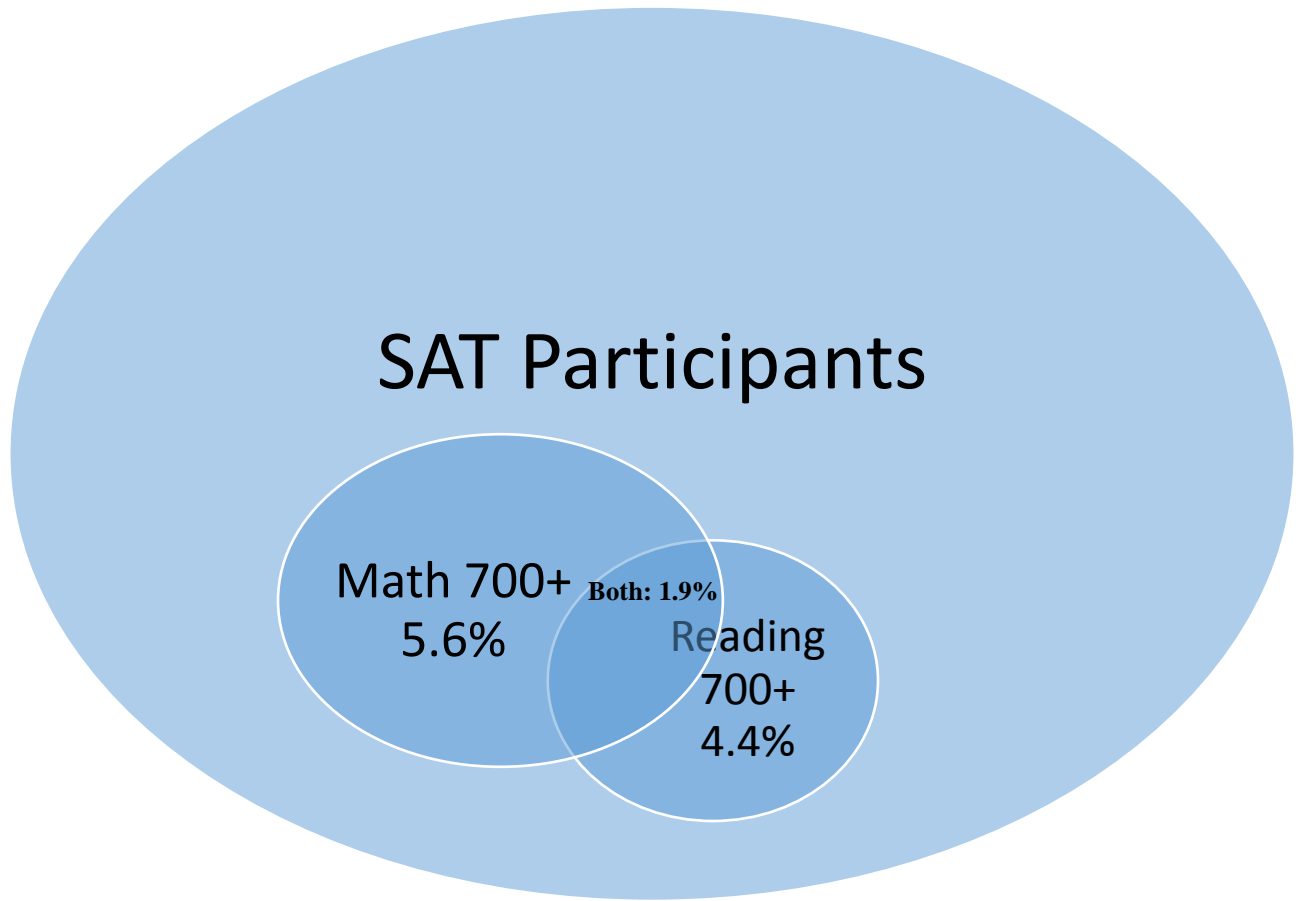
Therefore, I constructed the following equation to solve for his missing data:

$$0.054 * 0.36 = 0.01944 = y * 0.44$$

where y is the percentage of students who scored at least of 700 on the reading portion of the SAT. As it turns out from the equation, $y \approx 0.044$, or 4.4% of the total students. Thus, more of the analyzed students achieved a “top performers” score in mathematics than in reading.

Hacker’s comparison of the duel “top performers” may look dire, but once his numbers are *algebraically* analyzed, it is apparent that this is a mostly meaningless comparison.

Figure 1: Venn Diagram of Hacker’s Data



When Hacker argues that he believes the United States ought to abandon “advanced algebra,” he presents a case for a purely arithmetic method of educating students in mathematics—notwithstanding that, as Isaac Asimov (1961) observed, “algebra is just a variety of arithmetic” (p. 5). In his class, his students will encounter little, if not zero, algebra in assignments. Instead, he opts to provide students with data and asks them to operate on the values with arithmetic. His class appears to focus primarily on hypothetical situations, such as attempting to craft a modified method of measuring time so that our time system would be decimal. When he arrives at statistics, he provides students with Pennsylvania election data and asks them to consider what it means. While he claims that the class is a mathematics class, his background of political science emerges when he seems more interested in the political reasons

behind the results than how to calculate statistics beyond simple percentages. As it becomes apparent from examining his other statistical points, Hacker seems to present percentages as the most important element of statistical analysis. He says nothing about normal, binomial, or any of the other distributions, key elements of any basic introduction to statistics. Still, his class includes two powerful examples of historical mathematics. A major element of his class involves introducing students to a discovery based method of learning mathematics. Hacker provides an example using different ways of determining an estimate of π . Here, his students use different exploratory methods to find a rough estimate of π . Hacker incorporates historical methods, such as taking a piece of string with the same length as a circle's diameter and measuring the circumference of the circle using the string to create the estimate of $\pi \approx \frac{22}{7}$. The other historical example is perhaps even more sophisticated. In order to measure the area of the state of West Virginia, Hacker instructs his students to place evenly spaced dots on the paper and count the dots. He then has his students repeat this process with closer together dots, which creates a more accurate estimation. Although no algebra is explicitly used, Hacker is emulating the method of exhaustion, most famously used by Archimedes (DeSouza, 2012, p. 2). In addition to Archimedes, the method of exhaustion is often found in an introduction to Riemann sums and integral calculus. Thus, for all of Hacker's claims about how students do not need algebra, his class involves advanced mathematical concepts that are valid for all students.

Dr. Hacker often claims that students will never use the information taught in "advanced algebra." However, as Isaac Asimov (1961) observed, algebra is vital to developing the sciences (p. 132). In *Realm of Algebra*, Asimov explains how Galileo and Newton both used algebra to develop their theories on gravitation, while Cavendish utilized algebraic thinking to determine the weight of the earth. Although these are specific case of highly specialized careers, Asimov

explains that it is not necessarily the examples that lead algebra to importance, but how it has demonstrated that, when we utilize the thinking skills involved in mathematics, we discover new things about our world. The specific mathematics are not the important element, but the ways we were led to think were critical.

Educational Methods for Consideration

While Hacker's consideration of mathematics education contains helpful information, his insistence on lowering the bar for students marks a dangerous movement. If we wish to become more mathematically adept, lowering the standards makes little sense. Unfortunately, students are arriving in high school ill-prepared for even basic mathematics (Neild, Stoner-Elby, and Furstenberg, 2001, p. 32). In my own experience, my Algebra I students are largely below high school-standard for mathematics. I have some students who test at a 4th grade mathematics level that have been placed in an Algebra I classroom. Although Hacker's model of removing "advanced algebra" from the curriculum would assist these students in graduating, professionally I cannot find that lowering the standards is the correct option. Instead, I shall suggest utilizing an educational method that blends effective strategies from multiple educational schools of thought. Among the schools are constructivism, communicative mathematics, project-based mathematics, traditional American mathematics, and Japanese mathematics education. These environments demonstrate that it is possible to educate students in such a way that they are entirely prepared for a numerate life (Kelly et al 2013, p. 14-15). Under the hybrid method I will construct, I believe that students should be capable of handling higher standards while still learning effectively.

As a mathematics education movement, constructivism calls for the practice of only teaching concepts as built upon by previous learning and relying upon "no clearly unwarranted assumptions" (Pourciau, 1999, p. 721). Beyond utilizing previous learning, constructivism also emphasizes hypothesizing, communicating, and reflecting on mathematics (Davis, Maher, and Noddings, 1990, p. 2). In building concepts with constructivism, a student is not said to be learning until said student thinks about the mathematics and corrects any mistakes within the

process (Noddings 1990, p. 13). As observed above, these concepts are in line with OECD's definition of numeracy, since OECD expects students to demonstrate active reasoning skills in mathematics. Also, given that students tend to learn well from scaffolding, constructivism appears to be a helpful area from which to establish a teaching method. We see that there are values to admire about utilizing previous learning and limiting ourselves from assumptions.

At the same time, there are some problems with constructivism that arise from the perspectives some constructivists adopt. One perspective is the belief that all knowledge is constructed and that structures are either innate or the product of construction (Noddings, 1990, p. 7). This then implies that if we adopt this belief into our mathematics classrooms, then we are left with very few starting points. I cannot accept radical constructivism as my recommended teaching style. However, its emphasis on scaffolding and reasoning are certainly valuable elements to pull into a hybrid method.

Examining multiple schools of educational thought, Dr. Jo Boaler published her study of two British high schools in her book *Experiencing School Mathematics*. These schools, plus one American school, are also distinctly studied in *What's Math Got to do with it?* Each school carried its own brand of mathematics education. Although some mathematics scholars have attempted to discredit Dr. Boaler, her work remains a testament to the benefits of different avenues of study. Particularly in *What's Math Got to do with it?*, Boaler compares the three methods against each other. Meanwhile, *Experiencing School Mathematics* provides her readers with greater detail in the comparison of the original two schools.

At the American school (called Railside for the purpose of the study) Boaler observes that it is an urban school in California. Defying the stereotype of urban schools being low rigor and unsuccessful, Boaler discovered that the mathematics program at Railside appeared to set

students up for success. Instead of the *drill and kill* method (rote practice and memorization) that students often complain about, Railside's teachers worked together to form a new curriculum and teaching style that focused on teaching students how to represent the different mathematical concepts (Boaler 2015). With this method, the students would focus on one topic and study multiple ways of operating on it. Another key component of this method was communication. Students were expected to explain their reasoning to each other (Boaler 2015, p.59). Meanwhile, teachers communicated with each other to determine what was working in each class. Because of this focus on discussion, Boaler describes Railside's method as the *communicative* approach. This route is particularly powerful at encouraging students to consider what it means to practice mathematics. Since students are observing multiple methods of solving problems (Boaler 2015, p.67), they learn that there is not just one method that we can use when approaching mathematics. Therefore, these students engage mathematics by considering their techniques and representations. When confronted with a topic they may not have directly studied, their ability to approach topics from multiple angles provides the students an opportunity to express their mathematical reasoning skills.

Railside's communicative approach is an application of constructivism in its emphasis on discovery based learning. Both viewpoints encourage students to learn from a hands-on approach to mathematics education. However, unlike radical constructivism, the teachers still held specific conclusions they wanted their students to arrive at. Thus, the discovery and scaffolding element of constructivism remains in this system, but there is less of a focus on the personal side of this approach. Mathematical representation remains largely standardized and concepts are acknowledged to exist, even if the students don't recognize them at first. One example of this is when Boaler observed a class based on finding algebraic patterns in graphs of squares. The

patterns already existed before the students discovered them, but students were encouraged to explore the graphs to determine the mathematics of the pattern. Another interesting note about this particular Algebra I lesson was these students were given non-linear functions to solve. Said students would normally be completely unprepared for an introduction to non-linear functions without direct instruction. However, thanks to the thinking processes that communicative learning encourages, the students were prepared to handle these difficult topics.

Boaler's next school of study follows a similar path as Railside. Calling the school Phoenix Park, Boaler explains that although this is a predominately low-income school, the students there demonstrate a high level of learning. Like Railside, the school teaches by discovery. However, whereas Railside provides students with concrete examples to utilize for discovery, Phoenix Park is entirely project based. Students are provided with open-ended problems and asked to consider solutions to them. Another vast difference between Railside and Phoenix Park is that in the project based school, students have a variety of options to select for their projects whereas at Railside there is only one set of problems to work through. Thus, we see that Phoenix Park's method lends itself better to *differentiation of instruction*, which is the adjustment of instruction to meet the individual needs of students. Since the projects are open-ended, the students are encouraged to consider their own methods to approach the problems. Not only are the problems open-ended, but the students are given plenty of opportunity to choose how they want to approach problems. By allowing student choice, Phoenix Park gave its students the opportunity to realize that mathematics can be an engaging course of study. If students appear to be stuck or bored, the teachers are encouraged to suggest a new mathematical concept to the students.

Boaler reveals in *Experiencing School Mathematics* that for all the benefits to learning occurring at Phoenix Park, the students spend significantly more time off task than when compared with other student populations (Boaler 2002, p. 64). Because of the discovery based model, the teacher tends to allow students to work at their own pace for a portion of the class time before stepping in to intervene. However, for certain students, this leads to them choosing not to focus on their education. Although average growth was shown to be higher than the traditional instruction school, there are students who struggle when placed in this model and experience lesser or no growth (Boaler 2002, p. 75). In the Phoenix Park study, students who did not engage with discovery learning suggested that they would prefer a more structured approach with direct instruction. Thus, a strategic application of this method might prove the most helpful across a school. At the same time, Boaler also notes that she believes the students who refused to work in Phoenix Park's classes were students who most likely would disrupt other classrooms. As a side note, the disruptive students eventually adapted to the educational method, suggesting their struggles were related to maturity (Boaler 2002, p. 75-76). Allowing a variety of educational styles for students to choose from might allow the school to tailor its education to suit the population's needs. However, as a single strategy, the overall growth appears to significantly outweigh the time lost to students being off task and the disconnect from students who do not connect with the model.

On the other side of Boaler's *Experiencing School Mathematics* and *What's Math Got to do with it?*, the school Amber Hill presents a perspective on traditional mathematics education. Although the school is in England, the method of education is incredibly similar to most of the American education system. Like Phoenix Park, Amber Hill was a largely low-income school with otherwise similar demographics. Since the schools are similar in their student composition,

they served well to reflect the differences in the education method. At Amber Hill, students sat largely in silence as their teachers presented on mathematics. New topics were introduced via lecture and example to the entire class rather than individually, with homework serving as practice. Once the teacher was finished lecturing, the remainder of class would typically result in silent homework time.

Beyond the traditional education style at Amber Hill, students were also tracked throughout their middle school years and were placed in differing levels of classes based on perceived ability. This meant that students were arranged in classes based on similar skill with mathematics, yet Boaler found key issues with student learning. Although the top two classes at Amber Hill were comprised of the initially “brightest” students, she discovered that these same students eventually despised mathematics significantly more than their compatriots in the lower tracked classes (Boaler 2002, p. 161). Suggesting that the nature of the tracked classes led to unnaturally high expectations for students, Boaler claims that methods like Phoenix Park—where students are instead paired in mixed-ability groups—better prepared all the students for their mathematics education (Boaler 2002, p. 168).

Perhaps the other major significant different between Railside and Phoenix Park vs. Amber Hill was the students’ perceptions of mathematics. Students at both Railside and Phoenix Park expressed a belief that mathematics is like a language with many approaches to a single end (Boaler 2015, p. 59). However, at Amber Hill the presiding opinion was that there was for each mathematics exercise a single route to solve the problem, which confused students once they faced their exams (Boaler 2002, p. 109). In addition, Amber Hill students relied nearly exclusively on the power of memorization of formulae to approach their problems, claiming that if they did not remember a specific route from keywords that their only option was to panic

(Boaler 2015, p. 77). Meanwhile, the more reasoning focused approach at the other two schools encouraged students to first consider what the problem was asking. Examining the Standards of Mathematical Practice (SMPs) as written by the National Council of Teachers of Mathematics, it appears that, despite Amber Hill falling more into the traditional style of education, Phoenix Park and Railside both practiced these standards significantly more often than their counterpart. In particular, the SMPs titled “Reason Abstractly and Quantitatively” and “Model with Mathematics” were met by the focus on learning how to think about mathematics and the projects. Since the students were trained in how to reason with mathematics, their ability to approach unfamiliar concepts surpassed that of Amber Hill’s students.

When Dr. Boaler (2015) performed her study on Amber Hill and Phoenix Park, she intentionally chose schools whose students were of roughly similar socio-economic status (p. 80) to compare the long-term effectiveness of the educational strategies. Outside of the methods the schools used to educate their students, there was little difference between the schools. However, eight years after her study had finished, Boaler interviewed students from the study. She determined that the Phoenix Park students demonstrated upward socio-economic mobility, with 65% of the interviewed students achieving more professional jobs than their parents and only 15% with less professional jobs. This is compared with numbers from Amber Hill at 23% and 52% respectively (Boaler 2015, p. 80-1). Although their mathematics education is most likely not the sole cause of this difference, Boaler determined that the Phoenix Park students demonstrated more confidence and appreciation for mathematics than their peers from Amber Hill. Typically, the graduates from Phoenix Park would relate mathematics back to logical reasoning and problem solving, while the Amber Hill graduates would remain focused on quantities and numbers, claiming their education had nothing to do with post-secondary life.

Unfortunately, although mathematics is designed to teach students how to think critically, the solely lecture based method was not conducive to students developing their thinking abilities. These differences suggest that the training provided by the communicative and discovery methods better prepare students for a lifetime of appreciating mathematical reasoning.

Although the above statements paint a dire picture of the traditional mathematics method, there are still beneficial aspects of the teaching style. For some students, sitting and hearing the lessons allows students to better absorb the learning. Students at Amber Hill also demonstrated the capacity to remain on task longer and more consistently than students at Phoenix Park (Boaler 2002, 2015). In receiving instruction directly, the students were taught the methods exactly as they would be tested on their exams. Since students were shown the standard styles, they were more prepared for normal notation than if they had needed to determine their own. The key here appears to be communicating the information effectively and with the caveat that the students should also be taught to consider how the information was developed. Although I would not recommend solely relying on traditional instruction methods, there is clearly a place for them in any mathematics classroom.

Returning to my prior examination of constructivism, we can notice similarities between the discovery models at Railside and Phoenix Park and the tenets of constructivism. The projects from Phoenix Park fall in line with one of the stated goals of constructivist philosophers to allow students to visualize mathematics (Davis, 1990, p. 96). Since each project that Dr. Boaler describes relies upon a real-world scenario that the students ought to imagine visually, the students receive a context for their mathematics that allows them to consider how the math operates on physical objects. Likewise, where Davis (1990) claims that teachers must explain to students the reasons behind our use of mathematical concepts (p. 102), both Phoenix Park and

Railside engage with the questions “How?” “Why?” and “When?” while teaching mathematics to the students. This allows the students to connect with their math learning and understand that mathematics is not arbitrary, but instead a wondrous system that communicates information.

Although I have focused primarily on the United States’ education system with a splash of the United Kingdom’s, other countries have effective methods of educating their students. Of particular note, Japan boasts powerful mathematics classrooms. According to Shigeo Yoshikawa, member of Japan’s Ministry of Education, Culture, Sports, Science, and Technology, roughly 97% of Japanese students make it to 10-12th grade, where the mathematics education becomes increasingly strenuous (2008, p. 11). This focus on math begins before these higher grades, however. In the United States, we attempt to provide each subject with an equal measure of classroom hours. Japan, however, allocates the second most of classroom and lesson hours to mathematics of all the subjects in the Japanese schools from 1st grade through 9th, only being beaten by Japanese classes (Yoshikawa, 2008, p. 15). This is even after mathematics received a cut in hours both when Japan introduced *integrated learning* as a subject throughout school and when the country shifted to a 5-day school week. The emphasis on mathematics allows students to develop an appreciation for mathematics during the early years of education. In their method, something appears to be effective, since their students’ average mathematics score in PISA 2012 was 536, second out of OECD member nations. Examining how high and low numeracy scores are distributed in Japan, level 1 Japanese students are 11% of their student population while high achieving (levels 5 and 6) students are 24% (Kelly et al, 2013). From above, we recall that the United States’ student population includes 26% of the population at a level 1 numeracy and 9% at the levels 5 and 6 mark. In forging my hybrid method, I shall examine Japan’s effective methods and borrow portions of them for the end product.

As a culture, Japan emphasizes the value of supporting the collective or group above the needs of the individual (Iwama 1989, p. 73). Thus, we see some key differences between American education and Japan's. Since the success of the group is encouraged and emphasized, very few classes in Japan are separated into ability groups. Thus, most students in Japan will participate in the same mathematics classes. In the classroom, students of all abilities are encouraged to participate in the learning. Students who are struggling in mathematics are supported by the rest of the class and the teacher, with errors being worked through together by the class until the student understands the concept (Stigler, Fernandez, & Yoshida 1996, p. 240). This leads to students actively working together to develop the learning for the entire class. Since students care for the success of their peers, they should see a corresponding strength in group learning environments. While American culture is highly individualistic, encouraging students to support each other in their educational process is a worthy goal to establish.

At the heart of Japanese mathematics is a respect and adherence to the National Curriculum Standards. Japan's Ministry of Education publishes a list of acceptable curricula for all subjects that each follow the national content standards (Nagakoa, 2008, p. 143). At the same time, these content standards are the minimum standard expected out of all students, which allows schools to teach concepts beyond the initial standard should their students be prepared for them (Yoshikawa, 2008, p. 19). For the Japanese mathematics classroom, the textbook is everything. Unlike American textbooks, however, Japanese textbooks are small and light. The American textbook has every lesson that the students could potentially learn in the entire year of mathematics learning with large quantities of examples and exercises for the students. Meanwhile, the Japanese textbook is more concise. Students are expected to learn all the lessons and concepts within the textbook (Peterson, 2008, p. 210). These thinner textbooks allow

students not to feel as overwhelmed by the flow of mathematics. Similar to the learning at Phoenix Park and Railside, the Japanese textbooks develop themes through an emphasis on questioning strategies. Rather than provide direct examples of every potential concept, the students are encouraged to think their way through lessons, using class problems to build the necessary bridges between prior and current learning (Peterson, 2008, p. 216). By starting their lessons with questions instead of examples, the Japanese teachers encourage their students to engage with the learning.

We also see a major difference in the American and Japanese teacher's manuals for textbooks. In the United States, we have massive teacher's manuals, filled with extraneous information about the topics within. Particularly for mathematics, teachers regularly receive multiple books to use as part of the manual. Lee and Zusho (2002) examined two American elementary mathematics curricula and discovered one included seven different books for the teacher to use while the other included a main teacher's manual that weighed more than 5 pounds on its own (p 70-1). While it is admirable that so much information is provided, there are a few significant problems. On the one hand, the overabundance of information provided to the teachers tends to result in vital information either being overlooked when the teacher reviews the material or being buried under extraneous information. Although the Japanese manuals are significantly lighter and smaller, they contain one key element that the American manual sorely requires: responses to common student questions/misconceptions (Lee and Zusho, 2002, p. 84). In the place of the common student, the American manuals provide insight on how to reach the fringes of the classroom. Although this is valuable, understanding how also to reach the general population is vital to developing widespread numeracy. If the teachers do not understand or

anticipate common confusions, their ability to assist students in developing their numeracy will be crippled.

Between the Japanese model and the different American/English methods, we can observe how the Japanese follow a similar style as Boaler's Phoenix Park and Railside. All three schools follow a discovery-based model to encourage students to learn how to operate with mathematics. Through this method, it appears the students on average develop their numeracy more thoroughly than others. In all three studies, the students were very adept at developing why methods they chose were effective or useful. They have built their reasoning methods and have prepared themselves for the process of finding difficult solutions. At the same time, students in this model apparently demonstrate higher scores when examined, whether this is Japan's significantly higher PISA scores than the United States (Kelly et al 2013), Phoenix Park's students outpacing Amber Hill's top classes in mock end of year exams (Boaler 2002, p. 115), or Railside's massive mathematics growth compared with other local schools (Boaler 2015, p. 66). As a whole, each of these surveyed models represent success stories for discovery based learning. While the discovery model is not effective for some early teen boys (Boaler 2002, p. 60), it is effective enough for the majority of student populations that given augmentation with direct instruction each student should find reason to engage in learning. Even the boys who struggled with discovery learning in Boaler's research benefited once the teacher supported their learning (Boaler 2002, p. 62). We see that rather than rely upon directly lecturing to the students, all three of these models encourage use of practice using real-life situations (Peterson, 2008, p. 210, Boaler, 2015, p. 69) above examples to teach students new concepts. Particularly when examining the results of Phoenix Park and Amber Hill, the students from Phoenix Park cited learning about the real-life practicality of mathematics when asked about the value of their high

school education (Boaler 2015, p. 81-2). These similar success stories suggest that the educational techniques applied within these systems encourages student success.

Although the three non-traditional teaching methods often reflect each other, there are critical differences that allow Japan to vastly outperform American education systems. One area in which Japan excels in the discovery model and where even Phoenix Park and Railside's students could seek improvement is the act of productive struggle. In normal American mathematics classrooms, students presented with an impossible to solve problem would struggle with it for only 30 seconds on average before they gave up on the work. On the other hand, the Japanese students took the problem and attempted to solve it for a full hour and would have continued engaging with it if class had not ended (Spiegel 2012). The key here is the cultural differences between Japan and the United States. Whereas struggle is often seen as weakness and a lack of mental preparedness in the United States, Japanese teachers and parents welcome struggle. To them, the struggle is a sign of learning and developing. After all, if a student was not struggling with something, it obviously would not need to be taught. Struggle in Japan is used to demonstrate that students are engaged in the topic at hand. This directly translates into a respect for students who are having difficulty. I am impressed by how Japanese teachers approach group demonstrations. Unlike American schools, where we encourage our strongest students to share their work to the whole class, the Japanese classroom encourages the struggling students to demonstrate their methods. Since these students are normally the ones who are not quite grasping the content, their approaches tend to fall short. However, the demonstration is not to embarrass the student, but to allow the struggler to learn from the entire class. In the example that Spiegel (2012) reports, the students were respectful to their peer who had not yet grasped the method of graphing a cube. Instead of teasing their classmate for his struggle, they would answer that his

method was not correct when prompted by the teacher. Eventually, the student successfully constructed his cube for the whole class to see. If this exchange occurred within a typical American classroom, the student would likely not encounter a friendly and respectful attitude from the rest of the class. Most American classes would immediately begin attempting to correct any mistake that they find. This harms their classmate, since now the student loses the chance to productively struggle.

We have now examined three different educational methods: constructivism and the schools at Railside and Phoenix Park which applied a form of it, traditional direct instruction from Amber Hill, and the Japanese educational system. Each of these systems has demonstrated its strengths. In an effort to improve the American mathematics classroom, we should now consider how we might adapt elements from each of the techniques into one classroom. While the method I have designed can be applied to classrooms beyond algebra, the severe need in the United States for a strengthened approach suggests I ought to focus at the common starting point for high school mathematics—Algebra I.

Hybrid Teaching Method

Based on my analysis of the above theories of education, it appears that a new method could be constructed utilizing the strengths of these diverse educational systems. Each observed system brings a new perspective to teaching mathematics. While we might argue that individually the systems successfully contribute to student learning, teachers always are looking for more powerful educational methods. In order for this upcoming hybrid model to function, both the teacher and the students must participate in the system. If the class itself refuses to accept a new approach, the teacher may face insurmountable resistance to the approach. As I continue in this section, I will suggest recommendations of methods that may be appropriate to adapt from the different methods while elaborating on each method. By drawing from these positions, the hybrid method will become apparent.

As we have explored above, educational constructivism claims that all knowledge is constructed by individuals (Noddings 1990, p. 7) rather than established by an outside authority. We should note that mathematics uses constructivism differently than the educational branches. For this exploration, we shall only focus on the education version. Each individual constructing knowledge translates into a constructivist classroom relying heavily upon prior learning. Because knowledge is either innate or the result of previous learning, constructivist classrooms typically invoke scaffolding, encouraging students to base their new learning on the more basic building blocks. Obviously scaffolding has been accepted as a vital component in education, though the increased emphasis is certainly welcome. Following the theme of all knowledge being constructed, Noddings (1990) suggests that even erroneous constructions serve an educational purpose in mathematics, claiming that these mistakes, when corrected, serve to establish acceptable mathematical learning (p. 13-4). How we approach the theories of how mathematics

is conceived in the brain obviously impacts the methods we use to educate our students. If we consider that students will construct their methods, then they ought to believe their method is valid, useful and true (Confrey 1990, p. 111). Following this example, in the hybrid model I am approaching, students should not be forced to employ specific methods in solving problems. The teacher should certainly demonstrate, instruct, and otherwise educate students in mathematics. However, each student will find a particular methodology that works best for each case. In short, the overarching theme of a constructivist viewpoint of mathematics education is that the role of a teacher “is to establish a mathematical environment” (Noddings 1990, p. 13). Part of this mathematical environment is ensuring that the teacher does not allow bad mathematics to grow in the students. The teacher must have a “specific agenda” when presenting a lesson, even if students are to discover properties of mathematics (Confrey, 1990, p.122). Allow the class to approach scenarios according to their own methods, but be prepared to correct misconceptions before they permanently set. Thus, the students construct their own knowledge while the teacher, the authority in the classroom, serves as an overseer.

As an example of establishing a mathematical environment as an overseer, when teaching algebra students how to solve a system of equations, typically three methods are taught--graphing, substituting, and eliminating. For each lesson, the students should be encouraged—perhaps required for the day—to practice that approach. However, once the instructional period for the different techniques has passed, the students should be encouraged to find whatever strategy was most effective for them and allowed to choose as they please, even if it might be guessing and checking. While each approach occasionally is less optimal than others for certain situation, the ability to select their own methods encourages students to consider the applicability of mathematics to different situations. Instead of testing students on specific methods of solving

systems, allow the students the opportunity to determine their own path. Also, while designing questions, teachers can design questions such that the structure might lead students towards a more optimal strategy, but the students should still be allowed to select their preferred methodology. The key here is to express that there are many approaches to problems in mathematics, each with its own strengths and weaknesses. By demonstrating the models and allowing students to experience them, the teacher has created a space where mathematics is learned, practiced, and applied while still allowing space for discovery.

While mathematical constructivism provides valuable inputs to an educational model, radical constructivist theories can present obstacles to a productive education. For example, there is a school of thought in radical constructivism that insists a statement cannot be true until it is confirmed via observation/experience (Goldin 1990, p. 31). Thus, a radical approach to education implies that teaching itself is insufficient to establish the validity of any knowledge. While this heavily encourages discovery-based learning, it certainly can hinder the capability to establish facts. We also encounter the situation of radical constructivism claiming we can never determine if our “own knowledge is ‘the same as’” someone else’s (Goldin 1990, p. 34-5). Thus, the mathematical models taught by the educator are potentially useless. Since the learner must observe or experience the information, a direct instructional approach is rejected within a strict application of radical educational constructivism. Constructivism brings with it plenty of benefits, though these pitfalls must be considered.

Dr. Jo Boaler’s examination of two educational constructivist schools, Railside and Phoenix Park, indicate that a discovery and project based method powerfully encourages students to view mathematics as a useful tool. Based on her interviews with ex-students from the schools where the students cited their mathematics education as relevant to their professional

lives (Boaler 2015, p. 81), it appears that the focus on project-based education in those two schools demonstrated the value of mathematics in future life. In her examples of lessons that the students received, there was a heavy emphasis on life context in the mathematics—how the math related to the situation described.

The project based methods of Phoenix Park and Railside serve as helpful examples of well-designed and operated constructivist education systems. In both schools, the students experience mathematics via experience and modeling, either by Phoenix Park's project system (Boaler 2015, p. 69-70) or Railside's discovery-based instruction (p. 59). When examining the strengths of these constructivist systems, we can observe that it appears students vastly outperform their peers in other, more traditional systems. Railside's students surpassed two other studied schools within two years after beginning behind in mathematics learning (Boaler 2015, p. 66). We also can note from the students' answers in interviews with Boaler at Railside that students were taught how to learn in the constructivist model, claiming success in math classes required "asking good questions, rephrasing problems, explaining ideas, being logical, justifying methods, representing ideas, and bringing a different perspective to a problem" (p. 67). Each of these approaches grants students the opportunity to develop beyond a "rules based" understanding of mathematics and reach a conceptual understanding of the topics.

Revisiting our example from above of solving systems of equations, an approach similar to Railside and Phoenix Park's allows students to build concrete understanding. To begin the lesson, the teacher would present a situation based in a real-world context. For this example, we shall assume the project is modeling total revenue based on the number of sales of two different products. The students would then be provided with the total number of sales and asked to determine how many of each product was sold. Note that our teacher has not directly informed

the students of any of the three methods. The students should be allowed to take time to attempt to solve their system of equations. If an answer is reached, then the students should be asked to explain what their answer means. The nature of this scenario allows the students to work towards discovering any of the three methods during the first lesson. If students appear to be struggling, the teacher can observe which method the students' work most closely matches and then guide the students towards that specific method. For example, a student who has begun to graph the lines made by the system would most certainly benefit from being taught to look for the intersection of the lines. Meanwhile, a student who has considered trying to determine how to calculate the number of a single product using the total minus the other product is certainly ready to hear about substitution. The key is for the first day of learning to be largely student directed and allow students to experience their learning in their own ways. By taking a day to allow for exploration, students are allowed to develop firm understandings by experiencing the mathematics for themselves.

Traditional American/English mathematics education—while often disparaged—brings certain benefits to mathematics education. Even in less traditional classrooms, such as project based discovery classes, students still require direct instruction for difficult topics. Particularly when confronted with the reality of mandatory state testing, certain procedures and topics must be taught to allow students an opportunity to pass the tests. Even in Phoenix Park, Boaler (2002) observed that the school adjusted its educational approach to a more traditional method for about a month before the major testing began (p. 80). This suggests that even though the project method generally served the interests of the teachers and students, the traditional method better fits a standardized testing environment. We can see that traditional classrooms are also

significantly more structured than their counterparts. In my hybrid model, traditional lecture and presentation based teaching follows an initial foray into discovery learning.

Continuing the example of systems of equations, after the day of exploration and construction of knowledge, a traditional instruction lesson can take place. Since students now have begun to develop for themselves their own methods, a series of lessons can now take place synthesizing the different approaches that hopefully developed. One key feature here is to continue using models for the examples. Given a real-world context, we can begin to bypass the common refrain from students of, “When will I ever use this in the real-world?” If they are aware of how the concept—in this case systems of equations—is used in a variety of contexts, hopefully students will begin to understand that their learning applies to their lives regardless of the career path they eventually will select. From above in the discovery portion, I have already demonstrated that systems of equations can model how many of specific products were sold if we know the total profit and number of sales. We can also consider situations where events need a specific number of chaperones or security members depending on the number of attendees. If we expand our education topic to include exponential or quadratic functions (which currently exist at the tail end of Algebra I curriculums), the applications continue to develop. Thus, systems of equations can help our students realize what banking plans serve their interests better over time.

As an example of scaffolding using the systems of equations series, a lesson could be developed as a follow up to systems of equations once exponential functions have been taught. This lesson continues the above theme of investment plans. In this discovery lesson, the teacher will introduce students to two investment plans, one simple interest and the other compound interest. For best results, select a system so that there are two solutions to the system, such as

$$\begin{cases} y = 100 * (1.02)^x \\ y = 5x + 50 \end{cases}$$

This system models two investment plans, one with a \$100 investment with 2% annual interest, the other with a \$50 investment and a 10% simple interest. At this point in the students' education, they should be familiar with the three methods of solving systems of equations. More than likely students will begin to settle on graphing to solve this system once they realize substitution and elimination prove largely unhelpful. As I mentioned above, this system has two approximate solutions, (19.32,146.62) and (70.02, 400.08). After rounding to the nearest whole numbers for x, we see that the more profitable investment plans switch at twenty and seventy years. While the seventy-year data may seem useless to the average person outside of corporate investments, knowing how to find where different investments become more valuable is a practical skill for any citizen.

After examining educational methods from the West, I will now turn my attention to the Japanese model of education. Immediately, one strength of the Japanese school system is their approach to textbooks. Since Japanese textbooks are significantly smaller than comparable American books (Lee and Zusho, 2002, p 70-1), students and teachers are often less intimidated by the quantity of material they are required to cover each year. By using multiple small booklets, this hybrid method can avoid the struggle of “forgetting” to bring the books to class for studying and working. The more focused approach to lessons, courtesy of the emphasis on discovery learning in this model, can also be applied to the smaller books. Student books in this model would primarily focus on the application of topics examined in class, while project ideas and other introductory lessons could be relegated to the teacher's materials. Since students are encouraged to construct their knowledge of the topics, there is less of a need for pages of

structured formulae. Instead, the written portion of instruction would be developed by the students themselves, using the notation that they develop as the practice solving real-world situations.

When considering Japanese methods of education, we ought to consider how Japanese instruction focuses itself differently from American education. We have already observed that while Japan is incredibly adept at preparing teachers for the common classroom and student experiences, American teacher preparation focus primarily on outlier populations within the school system (Lee and Zusho, 2002, p. 86). Although I have claimed that focusing on the standard classroom environment should be a priority, it is still important to examine our less common students and be prepared to develop their numeracy. Since I am including elements of constructivism in this model, it is important to remember that even in the discovery learning section, we should be prepared to correct misconceptions before they become standard practice for students (Confrey, 1990, p.122). While practice makes perfect is a common saying, practice makes permanent may be a better phrase. Therefore, in the accompanying teacher's materials for a hybrid program, common misconceptions ought to be the first focus on student development. At the same time, the American focus on outlying students prepares teachers for when they inevitably teach a student who falls outside the norm. Each student brings an important perspective, especially during the discovery and exploration lessons.

Beyond the element of textbooks and teacher's manuals, the Japanese model, particularly for elementary mathematics, brings plenty of promise to all levels of math education. In America, classes are heavily focused on individual work and success. Meanwhile, Japanese culture focuses on the group (Iwama, 1989, p. 73). This directly leads to more collaborative learning and classroom participation. One result of near universal engagement, we see greater

success in approaching problem solving, applications of methods, and mastering abstract extensions of the topics (Stevenson, 1989, p. 89). We also see from Stevenson's (1989) research that Japanese schools spend roughly 87% of class time on academic activities, while his surveyed American classrooms used 64% (p. 94). I conjecture that the focus on group effort and learning has a direct influence on this difference in learning. Since Japan emphasizes the collective, it seems apparent that a teacher would provide extra focus on academics during class time. This is especially valid since class allows the teacher to reach the entire group of students. Clearly, attempting to educate the collective plays into Japan's strengths.

It is important to note that Iwama (1989) specifically cites that that Japan's centralized and monocultural system is what allows their group education to remain powerful (p. 76). Although American culture is incredibly diverse, I still believe emphasizing group success would provide a mighty boon to American education. Most careers require employees to cooperate with their peers. While we are largely an independence-driven culture, we can still begin to inform students of the role collaboration plays in everyday life. By adapting the Japanese collective focus in mathematics classrooms, we can develop a cooperative spirit in our students that not only will lead to stronger class participation and scores, but also to more productive citizens once they graduate. If students can be taught to care about the success of both their peers and themselves, I believe mathematics education will vastly improve.

Despite all the benefits and strengths I have described in my method, it is important to acknowledge the weaknesses of the model. Most glaringly, this method is vulnerable to slowing down instruction. Since the ideal lesson sequence in this model requires taking time to practice discovery before direct instruction, classes in this model will likely fall behind faster-paced classes. Thus, although the method allows students to cover individual topics in greater depth

than their peers, there is a strong chance they will be exposed to fewer total lessons. Although the slower pace can be a detriment, a consequence of this slower pace is that we focus on constructing a thorough understanding of each topic we study. Discovery learning also involves a sizable investment into productive struggle since students are learning by attempting to solve problems they have not necessarily been directly taught yet. Based on Western cultural practices, there is the danger of our propensity to abandon difficult lessons as I have discussed above (Spiegel 2012). The issue of quitting is not solely relegated to elementary school, however. Boaler also noted that students from the traditional classrooms at Amber Hill typically would assume that they should spend at most two minutes on any math problem before determining it was not worth their time (Boaler 2002, p. 33). Clearly from childhood to young adulthood Western students reject struggle as an accurate measurement of learning. Translating these two issues into a discovery learning classroom, there is the danger of students quitting on a new problem before they have truly begun the hybrid learning process, since it requires significant learning investment. To counteract this tendency to reject productive struggle, a teacher in this method must take time at the beginning of the school year to establish the practice of productive struggle, perhaps by issuing both impossible and solvable problems to the class. Since the rejection of struggle as a sign of progress has become ingrained into our culture, we need to overcome the stigma.

As I focus on the use of mathematics outside of pure academics, it is essential to provide examples and practice using practical applications. By using project-based learning for the first step in exploring a new topic, this method encourages students to consider how they could use mathematics in their careers. Outside of the initial learning experience, examples and problems in our smaller textbooks should almost universally be centered in real-world contexts. In an

effort to meet the Common Core's requirement that students reason abstractly and quantitatively and model with mathematics (Council 2010, p. 6-7), my model heavily emphasizes the practice of creating mathematical models. Armed with the capability to apply mathematics to a variety of situations, students can reason with a wider variety of topics. Although my model largely focuses on concrete examples with little emphasis on the abstract, it is necessary to consider that abstract thinking is largely undeveloped in high school students' brains. Throughout mathematics we already teach concrete examples before exploring the abstract and generalized forms. Before we teach algebra to students, we build the structure of arithmetic that will define the entirety of algebra. Isaac Asimov claimed "algebra is just a variety of arithmetic" (p. 5) in his exposition *Realm of Algebra*. Thus, we have precedent for focusing on the concrete to reach the abstract. This practice does not end in high school, however. In integral calculus, we teach a variety of Archimedes' method of exhaustion to solve Riemann sums before developing integration.

In my student teaching this year, I have discovered what I believe is a nearly perfect example of a curriculum that could apply my teaching method to all mathematics classes and especially algebra. Bridges to College Mathematics (BCM), a remedial, college preparation class designed by college professors, high school teachers, and specialists, emphasizes "career and college readiness" (OSPI 2017). It focuses on a combination of discovery learning, direct instruction during lessons, and application to real-world situations. Based on the information provided by the Washington Office of Superintendent of Public Instruction (OSPI), the course is structured and designed for students who scored a level 2 (passing high school, not ready for college) on their Smarter Balanced assessment. Typically, these students have struggled with standard approaches to mathematics, with the average student being one who has taken Algebra II and either failed or struggled in the class (OSPI 2016, p. 11). While the profile of the typical

BCM student represents a struggling math learner, I have found the approaches within its curriculum and a modified version of my method still support the students in my classroom who qualify as above the level of the class. Since this class and its design have assisted each of my students, I believe BCM and my method could easily apply to any mathematics classroom.

The methods and structure within BCM map nearly perfectly to my hybridized approach. Since each lesson incorporates a form of mathematical modeling, it allows learners to identify the application of mathematics to their everyday life. Each lesson provides opportunities for discovery learning using multiple approaches. Some lessons are hands-on, such as building a slingshot and graphing the arc of a launched gummy bear as a precursor to quadratic functions. Others are purely mathematical. The course prepares students for logarithms using a mathematical approach, with the discovery learning involving evaluating how many years it would take a landfill to reach capacity if the trash is growing exponentially. Following either of the discovery approach methods, the structure of the class develops into a directly taught portion where specific concepts are taught by the teacher. For example, the quadratics unit immediately introduces the different notations for a quadratic equation while the logarithm section mathematically defines a logarithm for the students. These direct instruction sections are always followed with additional application projects where students can practice applying the newly taught techniques to a variety of real-world contexts. For each unit, there are multiple occurrences of this pattern of education. By establishing a clear and consistent pattern, students are better prepared for the process of learning mathematics.

While BCM is incredibly effective at instructing students in mathematics, it does have a significant weakness that must be addressed if it were to be adapted to an algebra curriculum. Because of the emphasis on deeply focusing on specific concepts, students do not have the

opportunity to learn the breadth of mathematics they would normally learn in a standard algebra class. For example, the course does not fully explore how to operate with the rules of exponents, nor the rules of logarithms. It also completely ignores trigonometry, which is a sizable portion of the Common Core function standards. Although an Algebra I course would not cover these topics, the structure of BCM suggests that this would need to be addressed in a general Algebra I classroom to ensure the class meets the Common Core State Standards. Adjustments to what topics receive additional time would be necessary to adapt the course. With the techniques of BCM and an analysis of how best to address required standards, my method would serve the algebra curriculum well.

The hybridized method at this point is designed to involve discovery learning and traditional instruction to direct students towards remembering mathematics for the rest of their lives. For an algebra class to incorporate my hybridized method, it will require the instructor to consider appropriate projects for students to engage with at the start of each unit. The discovery learning is the cornerstone to the entire method. As Boaler (2002) determined in her research, while students from a discovery model education were concerned they might not readily identify required notation on exams, they had absolutely zero qualms about being able to apply their methods to any of the problems they might face (p. 115). Based on the above and other findings from Boaler, Davis, Maher, and Noddings, constructivist education approaches lend themselves well towards assisting students in developing the confidence and ability to “formulate, employ, and interpret mathematics in a variety of contexts” (Mathematics Expert Group, 2010). Providing students with the opportunity to discover their capability to apply mathematics to multiple contexts lends itself towards fostering healthy numeracy. Still, the students’ concerns in a purely discovery-based method deserve addressing. Thus, after each project there should be a lesson

where the teacher presents common approaches for representing the recent topic. When we follow the projects with a lesson on standard notations of concepts, we address the concern students expressed about being unable to identify necessary notation. Each instructional element has its place in the structure of the hybrid method. Discovery and projects encourage students to interpret mathematics in a new context while direct instruction confers standard methods of employing the concepts. Once we have incorporated the two initial instructional approaches, it is important to establish additional applications of each concept. Here, the algebra class should be encouraged to solve real-life situations using the new information. While discovery learning plants the seeds of understanding how mathematics is applied to situations, observing additional contexts solidifies students' comprehension of the applicability of mathematics to the variety of contexts. We see from Pape and Tchoshanov (2001) that students learn best when exposed to a combination of algebraic and visual (such as projects or geometry) instructional approaches (125). Based on their study, utilizing a variety of methods to teach algebraic content leads students to develop mathematical flexibility by training them to consider different approaches to mathematical problems.

Throughout my discussion of algebra, I have focused on applying algebra to life. Thus, I am claiming we ought to teach applied algebra in high school instead of pure algebra. Although the purely general representations and formulae within algebra serve valuable purposes in further mathematics education, students in high school struggle with cultivating numeracy with only abstract concepts (Pape and Tchoshanov 2001, p. 123). Since the adolescent brain is under-prepared for abstract learning at the start of high school (Dumontheil 2014, p. 62), it is important to utilize concrete examples and contexts, such as the projects at the beginning of my method. As the students mature in their critical thinking skills, they will be more prepared for less concrete

representations. Especially for the first math class of high school, however, abstract concepts typically leave students only able to use formulae they were directly taught without adjusting to new concepts (Pape and Tchoshanov 2001 125). By adapting my hybrid method from Boaler's studied schools, Japan's school system, constructivism, BCM, and direct instruction, I have gathered successful practices from a variety of sources. Because of the success of Railside, Phoenix Park, and the Japanese schools' students at understanding the value of mathematics, following a more applied path serves to assist students in connecting with mathematics across a variety of contexts. Maintaining the structure of a direct instruction component, meanwhile, allows the students the opportunity to recognize common notation and methods so that they can test effectively.

Since this method is being applied to the American education system, it is important to consider how it interacts with the Common Core State Standards (CCSS). As Washington is one of 42 states to adapt the CCSS for our educational system, the theory I have constructed should fall under its domain. The CCSS has two components to consider, the content standards and the Standards of Mathematical Practice. While the content standards are certainly important, this method is largely concerned with how we teach mathematics. The content standards are best addressed by the order the teacher directs class projects through. Thus, we can set aside the content standards for this consideration. Instead, we should focus on the Standards of Mathematical Practice (SMPs). The SMPs are as follows:

- 1) Make sense of problems and persevere in solving them
- 2) Reason abstractly and quantitatively
- 3) Construct viable arguments and critique the reasoning of others
- 4) Model with mathematics
- 5) Use appropriate tools strategically
- 6) Attend to precision
- 7) Look for and make use of structure

- 8) Look for and express regularity in repeated reasoning (Council of Chief State School Officers 2010, p. 7-8)

From these eight SMPs, we see the structure of how the CCSS expects students to approach their mathematical learning.

Walking through each SMP, we can demonstrate that the hybridized educational style matches every standard. Fortunately, some of these standards are met by the same processes. By incorporating projects at the start of each sequence of lessons, the method allows students to follow SMPs 1 through 6. Obviously, since the projects involve modeling a real-world concept, SMP 4, “Model with mathematics,” is met. Likewise, depending on the topic of choice, students will be encouraged to utilize any available and useful tools, meeting SMP 5, “Use appropriate tools strategically.” For SMP 1 “Make sense of problems and persevere in solving them,” and SMP 3, “Construct viable arguments and critique the reasoning of others,” this approach is designed to incorporate group work, following the Japanese model of supporting the collective (Iwama 1989, p. 73). Thus, students need to be able to communicate their thought processes to each other when asked about what they are doing with their projects. Since the best projects are more open ended, SMP 1 is also met by students pursuing their answers and theories through the uncertainty of the unfamiliar. As the project process continues, SMP 2, “Reason abstractly and quantitatively,” becomes apparent since students need to determine their own methods of approaching the problems. Given their previous mathematical learning, students in this method are expected to draw upon their reasoning abilities to identify potential solutions to the project. Meanwhile, SMP 6, “Attend to precision,” can be handled by the teacher’s actions. By monitoring student work, a teacher in this method can encourage precise approaches and direct lost students towards methods that will allow them to maintain precision. Since the discovery

learning component is anchored in a real-life context, the students' work on the projects should still reflect reality.

Continuing to the direct instruction component, the teacher can encourage students towards SMP 7, "Look for and make use of structure." Once the students are familiar with a standard representation, they can examine the prior project to identify the structure they were taught. If the student had been stuck during the project, then by learning the structure of the new concept the student can address any confusion from the discovery learning. The final step in my hybrid model, using the new concept to solve additional real-world-based problems, meets the final SMP, number 8, "Look for and express regularity in repeated reasoning." By this step in the educational process, students have already seen one project and the structure of the concept taught from the project. By practicing with additional real-world applications of mathematics, we can train our students to identify the common signs of when to apply the mathematical topic to a problem. The entire process encourages students to consider how to apply mathematics to multiple contexts, just as OECD defines numeracy (Mathematics Expert Group, 2010, p. 4). The application of each SMP in the development of student understanding of mathematics using the hybrid method has been demonstrated. Therefore, since we can apply and adapt the SMPs to it, this method should be acceptable within the CCSS.

Conclusion

Returning to the scores from PISA 2012, it is apparent that the American mathematics system needs to develop alternative instruction methods to improve our numeracy. Since we were ranked 26th out of 34 OECD member nations (Organization for Economic Co-Operation and Development 2013, p. 2), we clearly need to improve our students' scores. A major element of this involves American attitudes towards numeracy in general (Paulos 1989, p. 4). Given a hybridization of constructivist discovery learning's ability to draw student interest into mathematics and direct instruction's guiding structure, mathematics education would benefit from the ability to better demonstrate the applicability of mathematics, reducing the common student attitude of "when will I ever use this?" Hands-on mathematical projects grounded in reality and supported by focused instruction are the key to guiding students towards an understanding of mathematics. If we can reduce the number of students who determine that they despise mathematics before ever reaching an undergraduate mathematics class, we may be able to cultivate lifetime numeracy for our citizens. Numeracy is more than algebra and arithmetic; it is the ability to operate with mathematics in a variety of contexts and with multiple approaches. Given that this hybrid method complies with the Standards of Mathematical Practice, it allows American teachers to adapt it to their classrooms without fear of violating their mandated standards. While this method is yet to be formally tested in a classroom environment, its basis in proven successful mathematics educational practices—Phoenix Park, Japan, direct instruction, and BCM—suggests it ought to operate successfully with students in multiple classrooms, including the high school algebra classroom. Based on the evidence above, it appears that student numeracy would improve should this method be applied.

Appendix: Faith Statement

How does my faith define who I am as a mathematical scholar? Perhaps this is a difficult question to answer, though I shall attempt to satisfy the curious adequately. Although mathematics is a difficult subject to tie faith into, I believe that I can still honor God by the way I live while studying. Also, since I intend to become a teacher, there will be a time when I have to teach without being able to express verbally my faith to my coworkers or my students while at school.

As an educator, I am pursuing a discipline where I may have difficulty expressing my religious commitments. Because of the laws that have been written, my expressions of my faith as a teacher will be limited to my actions rather than speech unless I teach at a private Christian school. While I cannot directly evangelize to my students, by my example I hope they will see a reflection of Christ. I can also, when asked, explain that I am a Christian without violating most district policies. If my students see my example as both a mathematics teacher and a Christian, they may see the grace of God.

Unlike my pursuit of educational scholarship, my mathematics degree will have no such constraints on my expression of faith. However, math itself is difficult to tie into religion. I want to honor God by my studies, but as we observed in Christianity and Scholarship, math and religion are nearly as separate as can be. Still, I can respect the beauty of mathematics and realize that God is the one who gives me the knowledge to understand what is happening when I am working with equations and formulae. I also hold the belief that because God establishes order instead of chaos, the power of mathematics to communicate and demonstrate order in the world is a sign of God's role as Creator of all. I believe the model of faith and learning that best

describes my approach to mathematics is the “Value-Added” model, where the discipline is just the discipline and faith is the interpretation of the discipline.

As a Christian, I reject the more radical elements of constructivism, particularly the claims that knowledge is solely a result of our own intellect and there is no “way things really are” (Confrey 1990, p. 108). Since I accept that the truth of Christianity has been revealed to us by God, I cannot accept that we merely constructed the revealed word of Scripture on our own. Likewise, because I believe truth is absolute, this leaves me with no recourse but to accept that there indeed is a way that things really are. While there are incredibly valuable theories and discoveries that constructivism has brought to the world, I have determined to uphold my Christian faith before educational theories.

During my study at SPU, I have been learning interpretations and theories of how faith can interact with my scholarship. According to George Marsden’s book *The Outrageous Idea of Christian Scholarship*, I can draw inspiration from the Incarnation of Christ for my interactions with my students. I found his comparison to a pilot who uses radar, but could see her “tasks differently if she believes she is ultimately dependent on God and that she has a spiritual responsibility to her passengers” (Marsden 91). Similarly, as a math teacher, I will be teaching concepts that intrinsically have nothing really to do with faith, but I have a duty given by God to care for my students. The command to love is key here. I can show God’s love to my students by caring for their education and development.

Examining Boyer’s four functions of scholarship in his book *Scholarship Reconsidered*, I see myself called to the scholarship of teaching. As a teacher, I ought to ask myself “how can knowledge best be transmitted to others and best learned?” (Boyer 24). Obviously, as a math teacher, I should be trying to communicate the ideas and methods of math to my students in a

way that they can understand and appreciate. Drawing from my mentor teacher's example, the scholarship of teaching should help students develop an appreciation for a subject even if they struggle with it.

Expanding upon Boyer's ideas, I see Douglas and Rhonda Jacobsen's statement that "the primary task of scholarship is to 'pay attention' to the world... with a sense of focus, care, and intensity that non-scholars lack" (Jacobsen). Since I will be teaching my students, as a scholar I ought to pay close attention to them and note how they are doing. If I observe that some students are struggling, instead of ignoring them, I should give them the attention and care that they need to hopefully allow them to succeed and grow in their math abilities. I should not abandon students when they need help as that would be a failure of scholarship.

A key element of Conservative Baptist theology is an emphasis on Christian liberty, the doctrine that God provides us with the freedom to choose whether specific actions are permissible or not. This theology is largely based on Romans 6 and the entire book of 1 Corinthians. To live Christian liberty properly, one must have integrity. I must be able to correctly judge how I live my life as a proper example of Christianity. Abusing my Christian liberty in teaching would harm my limited ministry at the schools. Thus, I must exercise discretion and act honorably. In my scholarship, Christian liberty is harder to observe, since liberty largely pertains to deciding to partake or abstain based on personal convictions. However, the integrity required to make those decisions leads me to live according to what is right. In demonstrating that I am morally upright, my scholarship becomes more respectable and allows me to prove that my love for my students and fellow scholars is genuine.

As a Conservative Baptist math teacher, my scholarship will be based in my faith. Drawing from the three examples I have described above, I hope to honor God and inspire my

students to follow my example in life. Even if I cannot directly express my religion, I can live in such a way that my scholarship will be a powerful example to my students. In addition to the example I want to set for the students, perhaps my scholarship will be visible to my coworkers. If I encounter coworkers who do not particularly care for their students or consistently gripe about situations, perhaps I can engage their teaching culture and change the way some might work. If that is the case, then I will have fulfilled another part of my calling as a math teacher as I will have inspired other teachers to take up the scholarship of the Incarnation. As it keeps coming up in discussions of scholarship, integrity and love are key. Correction according to love, teaching according to love, and pursuit of a subject because of love are all key components of my scholarship. Meanwhile, without integrity, my example is useless. If I am to be a representative of Christ, then I ought to live in a way that reflects Him. A two-faced reflection, even if it loves, is no reflection at all. Thus, by living with integrity and love, I become a better example of my Lord and Savior, a better mathematician, and a better educator.

Bibliography

- Asimov, I (1961). *Realm of algebra*. New York: Fawcett Premier
- Boyer, E. (1990). *Scholarship reconsidered*. New York: The Carnegie Foundation for the Advancement of Teaching.
- Boaler, J. (2002). *Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student learning, revised and expanded edition*. London: Routledge.
- Boaler, J. (2015). *What's math got to do with it? How teachers and parents can transform mathematics learning and inspire success*. New York: Penguin.
- Cobb, P., Wood, T., and Yackel, E. (1990). Classrooms as learning environments for teachers and researchers. In Davis, R.B., Maher, C.A., and Noddings, Nel (Eds.) *Constructivist views on the teaching and learning of mathematics. Journal for Research in Mathematics Education. Monograph number 4*. Reston: National Council of Teachers of Mathematics
- Confrey, J. (1990). What constructivism implies for teaching. In Davis, R.B., Maher, C.A., and Noddings, Nel (Eds.) *Constructivist views on the teaching and learning of mathematics. Journal for Research in Mathematics Education. Monograph number 4*. Reston: National Council of Teachers of Mathematics
- Council of Chief State School Officers (2010). *Common core state standards for mathematics*. Washington: National Governors Association Center for Best Practices.
- Davis, R. B. (1990). Discovery learning and constructivism. In Davis, R.B., Maher, C.A., and Noddings, Nel (Eds.) *Constructivist views on the teaching and learning of mathematics*.

Journal for Research in Mathematics Education. Monograph number 4. Reston: National Council of Teachers of Mathematics

Davis, R. B., Maher, C. A., and Noddings, N. (1990). Constructivist views on the teaching and learning of mathematics. In Davis, R.B., Maher, C.A., and Noddings, Nel (Eds.) *Constructivist views on the teaching and learning of mathematics. Journal for Research in Mathematics Education. Monograph number 4.* Reston: National Council of Teachers of Mathematics

Davis, R. B., Maher, C. A., and Noddings, N. (1990). Suggestions for the improvement of mathematics education. In Davis, R.B., Maher, C.A., and Noddings, Nel (Eds.) *Constructivist views on the teaching and learning of mathematics. Journal for Research in Mathematics Education. Monograph number 4.* Reston: National Council of Teachers of Mathematics

DeSouza, C. E. (2012). *The Greek method of exhaustion: Leading the way to modern integration.* (Master's thesis). Retrieved from The Ohio State University Library (820036656).

Dumontheil, I. (2014). Development of abstract thinking during childhood and adolescence: The role of rostrolateral prefrontal cortex. *Developmental Cognitive Neuroscience*, volume 10, October 2014, (pp. 57-76.) Retrieved from <http://ac.els-cdn.com/S1878929314000516/1-s2.0-S1878929314000516-main.pdf?_tid=34e30d2a-2dc7-11e7-822e-00000aab0f02&acdnat=1493572106_add35db3e6447d6bf9e912bd8a632d6d>

- Garnick, C. (2009, July 21) "Logic, math prove to be the most failed classes for students" *The Western Front*. Retrieved from
http://www.westernfrontonline.net/news/article_40200e52-8b87-5ce3-86af-28d448c7e3b3.html>
- Goldin, G. A. (1990). Epistemology, constructivism, and discovery learning mathematics. In Davis, R.B., Maher, C.A., and Noddings, Nel (Eds.) *Constructivist views on the teaching and learning of mathematics. Journal for Research in Mathematics Education. Monograph number 4*. Reston: National Council of Teachers of Mathematics
- Greens, C. E., & Rubenstein, R. (Eds.). (2008). *Algebra and algebraic thinking in school mathematics*. Reston: The National Council of Teachers of Mathematics.
- Grouws, D. A. (1992). Handbook of research on mathematics teaching and learning. New York: The National Council of Teachers of Mathematics.
- Hacker, A. (2012, July 28). Is algebra necessary. *The New York Times*. Retrieved from
http://www.nytimes.com/2012/07/29/opinion/sunday/is-algebra-necessary.html?_r=1>
- Hacker, A. (2016). *The math myth: And other STEM delusions*. New York, The New Press
- Iwama, H. F. (1989) "Japan's group orientation in secondary schools." In Shields, J J Jr. (Ed.) *Japanese schooling: Patterns of socialization, equality, and political control*. (pp. 73-84). University Park: Pennsylvania State University Press
- Kay (2016, May 2). What's the most failed college course? This will surprise you. [Web log comment]. Retrieved from <http://www.smartstudentsecrets.com/whats-failed-college-course-will-surprise>>

Kelly, D., Xie, H., Nord, C. W., Jenkins, F., Chan, J. Y., and Kastberg, D. (2013) *Performance of U.S. 15-year old students in mathematics, science, and reading literacy in an international context: First look at PISA 2012*. Institute of Educational Sciences.

Retrieved from <<https://nces.ed.gov/pubs2014/2014024rev.pdf>>

Lee, S. & Zusho, A. (2002). Comparing Japanese and U.S teacher's manuals: Implications for mathematics teaching and learning. In G DeCoker (Ed.), *National standards and school reform in Japan and the United States* (pp. 67-92). New York: Teachers College Press.

Marsden, G. M. (1997) *The outrageous idea of Christian scholarship*. Oxford, Oxford University Press.

Mathematics Expert Group (2010, November 30). *PISA 2012 mathematics framework: To OECD, November 30, 2010*. OECD Program for International Student Assessment.

Retrieved from <<http://www.oecd.org/pisa/pisaproducts/46961598.pdf>>

(2013, November 12) "Mathematics graduation requirements" Retrieved from

<http://www.k12.wa.us/GraduationRequirements/MathGraduationRequirements.aspx>

National Center for Education Statistics. *Selected findings from PISA 2012*. Retrieved from

<https://nces.ed.gov/surveys/pisa/pisa2012/pisa2012highlights_1.asp>

Neild, R.C., Stoner-Elby, S., and Furstenberg, F. (2008). "Connecting entrance and departure: The transition to ninth grade and high school dropout." *Education and Urban Society* 40 (5), (pp. 543-569).

Noddings, N. (1990). Constructivism in mathematics education. In Davis, R.B., Maher, C.A., and Noddings, Nel (Eds.) *Constructivist views on the teaching and learning of*

mathematics. Journal for Research in Mathematics Education. Monograph number 4.

Reston: National Council of Teachers of Mathematics

Organization of Economic Co-Operation and Development (2013). *Country note: United States.*

Program for International Student Assessment. Retrieved from

[<https://www.oecd.org/unitedstates/PISA-2012-results-US.pdf>](https://www.oecd.org/unitedstates/PISA-2012-results-US.pdf)

Pape, S. J., and Tchoshanov, M. (2001). The role of representation(s) in developing

mathematical understanding. *Theory into Practice*. (pp. 118-127). Retrieved from

[<https://www.researchgate.net/profile/Stephen_Pape2/publication/249901059_The_Role_of_Representations_in_Developing_Mathematical_Understanding/links/54d2f69e0cf250179181009d/The-Role-of-Representations-in-Developing-Mathematical-Understanding.pdf>](https://www.researchgate.net/profile/Stephen_Pape2/publication/249901059_The_Role_of_Representations_in_Developing_Mathematical_Understanding/links/54d2f69e0cf250179181009d/The-Role-of-Representations-in-Developing-Mathematical-Understanding.pdf)

Paulos, J. A. (1988). *Innumeracy: Mathematical illiteracy and its consequences*. New York: Hill and Wang.

Pourciau, B. (1999). The education of a pure mathematician. *Mathematical Association of America*. 106 (8), 720-732.

Powell, J. J. (1929) *A study of problem material in high school algebra*. New York: Teachers College, Columbia University

Stigler, J. W., Fernandez, C., & Yoshida, M. (1996) Cultures of mathematics instruction in Japanese and American elementary classrooms. In T. P. Rohlen & G. K. LeTendre (Eds.) (1996) *Teaching and learning in Japan*. (pp. 213-247). New York: University of Cambridge.

- Spiegel, A. (Host). (2012, November 12). Struggle for smarts? How eastern and western cultures tackle learning. *National Public Radio*. Retrieved from
<http://www.npr.org/sections/health-shots/2012/11/12/164793058/struggle-for-smarts-how-eastern-and-western-cultures-tackle-learning>
- Stevenson, H. W. (1989) "The Asian advantage: The case of mathematics." In Shields, J. J. Jr. (Ed.) *Japanese schooling: Patterns of socialization, equality, and political control*. (pp. 85-95). University Park: Pennsylvania State University Press
- Tate, W. (2002). African American children and algebra for all. In S. J. Denbo. & L. M. Beaulieu (Eds.), *Improving schools for African American students: A reader for educational leaders* (pp. 147-157). Springfield: LTD
- Usiskin, Z & Willmore, E (Eds.) (2008) *Mathematics curriculum in Pacific rim countries—China, Japan, Korea, and Singapore*. Charlotte: Information Age Publishing.
- Wagner, S. & Parker, S. (1993). Advancing algebra. In P. S. Wilson (Ed.), *Research ideas for the classroom: High school mathematics* (pp. 119-139). New York: Macmillan Publishing Company