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Where's the Rigor? A Study of Direct Instruction vs. Inquiry-Based Learning in Math Education

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Contents

Introduction

Is direct instruction or inquiry-based learning a more effective way to teach mathematics? This question is the source of an ongoing discussion and topic of research that is permeating the world of mathematics education. In my honors project, I investigated this topic of research by focusing my study around the following question: How do direct instruction and inquiry-based learning curricula integrate rigor into their divergent approaches when presenting content to high school mathematics students, based on the expectation for rigor as defined by the Common Core State Standards? I explored this question by conducting an analysis of high school mathematics curricula that employ different teaching strategies. In order to narrow the focus of my analysis, I focused on two contrasting approaches to designing curriculum—direct instruction and inquiry-based learning. For the purpose of this project, I will define direct instruction as a traditional style of teaching in which knowledge is simply and directly communicated by the teacher to the students, and I will define inquiry-based learning as a non-traditional style of teaching in which students actively construct their knowledge through investigation.[1] *Table I* further illustrates the characteristics of direct instruction vs. inquiry-based learning.[2]

TABLE I: THE 3 DIMENSIONS TO THE DESIGN OF EVERY CURRICULUM

Direct Instruction	Inquiry-Based Learning
1) Research Content	1) Research Procedures
2) Audience	2) Participants
3) Teacher-Focused	3) Student-Focused

Mike Healey (2005)
Linking Research and Teaching: Exploring Disciplinary Spaces and the Role of Inquiry-Based Learning

In order to narrow the scope of my project even further, I selected the trigonometry unit in two direct instruction and two inquiry-based learning curricula to be the focus of my analysis. The curricula that I selected were chosen because of their alignment to the Common Core State Standards, which is an essential component of the methodology that I designed for this project.

[1] Healey, Mike (2005). “Linking Research and Teaching: Exploring Disciplinary Spaces and the Role of Inquiry-Based Learning.” *Reshaping the University: New Relationships between Research, Scholarship, and Teaching* (p. 67). New York City: Open University Press.
[2] Ibid.

My exploration of the different curricula begins with a literature review that examines different perspectives regarding how mathematics is being taught in the United States' education system. Particularly, this literature review examines contrasting views on inquiry-based learning, an argument for why it is important to differentiate math instruction and how to do it, and an analysis of how current-day curricula promote teaching math. The next section of my project includes a framework for my analysis of the direct instruction and inquiry-based learning curricula. Essentially, I evaluate how well these four curricula satisfy the expectation of rigor in mathematics instruction as outlined by the Common Core State Standards. Specifically, I analyze how each component of rigor—conceptual understanding, procedural skill and fluency, and applications—is integrated into the direct instruction curricula and the inquiry-based learning curricula. Then, I compile the results from my analysis and draw a conclusion that seeks to answer the question of where should mathematics educators go from here in regards to designing rigorous lesson plans for their classes. In the next section of my project, I synthesize the components of the different curricula into a five-day lesson plan on trigonometric functions. This lesson plan is intended to provide an example of how math instruction can be differentiated in a way that includes elements of both a direct instruction curriculum and an inquiry-based learning curriculum. The final section of my project is a reflection in which I evaluate my lesson plan in terms of its synthesis of different instructional strategies and its integration of the three components of rigor.

My research topic is significant because it addresses a serious issue in today's education system, namely, an absence of rigor in high school mathematics curricula as evidenced by low standardized test scores in mathematics as well as by students' lack of preparation for college-level math classes.[3] According to the High School Publishers' Criteria for the Common Core State Standards, mathematics education in the United States has not adequately prepared students for higher level classes in mathematics.[4] My project seeks to further investigate this issue by examining how different styles of curricula are structured and how they present the same learning standards in different ways. In the next section of this paper, I will highlight some of the influential research that has been conducted on direct instruction and inquiry-based learning in mathematics classrooms as a way to situate this project in the context of our public educational system.

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[3] National Governors Association, Council of Chief State School Officers, Achieve, Council of the Great City Schools, and National Association of State Boards of Education (2013). *High School Publishers' Criteria for the Common Core State Standards for Mathematics* (p. 2). Washington D.C.: National Governors Association Center for Best Practices and Council of Chief State School Officers.

[4] Ibid.

Background

Overview

My project addresses how mathematics is being presented in high school curricula with a focus on direct instruction versus inquiry-based learning. In order to provide context for my project and show its relevance in math education, I reviewed eight pieces of literature that will provide a solid background for the curricula analysis that I conducted. Two of them express varied perspectives in regards to the controversial subject of whether or not an inquiry-based approach to teaching mathematics is effective. Three of these pieces of literature identify various issues with teaching strategies in mathematics classrooms and how these issues are ineffectively and inefficiently meeting the different learning needs of students. The other three pieces of literature analyze and critique the process of teaching and learning mathematics in our present-day classrooms. In particular, these three pieces of literature focus on answering the question of why math teachers need to change how they are instructing their students. In essence, this literature review is intended to provide background information and establish the context for direct instruction and inquiry-based learning in math education.

Teaching and Learning Mathematics

In this section of the literature review, I will focus on the teaching and learning of mathematics in our present-day classrooms. In *What's Math Got to Do with It?*, Jo Boaler argues that our math education system is based on an objective that is full of errors. She believes that students aren't given the opportunity to experience "real" mathematics until they get to graduate school, which means that the vast majority of students will never experience "real" mathematics in our education system. Boaler's argument is based on the assertion that up until graduate school, students are only learning about the rules and tools that they will need in order to become mathematicians, but merely learning about the rules and tools is not a very engaging way in which to learn mathematics.

One significant issue regarding math education that Boaler discusses in his article is the level of support math teachers receive from their administration. For example, she describes a particular math classroom that he observed as a positively ideal learning environment because of how the students were actively engaged in collaborative learning; yet despite the success that was happening in this classroom, the teacher was told that she could "no longer teach in this way."[5] Instead of supporting the inspiring learning that was taking place in this teacher's classroom, the school administration chose to listen to a small group of parents who were insisting that direct instruction was the only way in which math could be properly taught.[6]

Another issue regarding teaching and learning of mathematics that Boaler focuses on in her article is the way in which students experience mathematics. As Boaler puts it, "Good students use strategies that make them successful – they are not just people who are born with some sort of math gene, as many people think."[7] However, it is a common notion in our society that people are either good or bad at math, and once students believe that they are bad at math, they lose confidence in their ability to understand it. Moreover, mathematics is oftentimes viewed in a negative light by people. As Boaler recounts, "When I tell people that I am a

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[5] Boaler, Jo (2008). *What's Math Got to Do with It?* (p. 2-12). London: Penguin Books.
[6] Ibid. (p. 12).
[7] Ibid. (p. 13).

professor of mathematics education, they often shriek in horror, saying that they cannot do math to save their lives."[8] This stigma associated with math has a lot to do with the way in which mathematics is being taught. Mathematics is viewed by many students as a boring and an uninteresting subject because they find their experience in math classrooms to be unengaging.

Issues in Mathematics Education

In *A Mathematician's Lament*, Paul Lockhart expands on this argument that mathematics instruction fails to be engaging. In his lament, Lockhart provides us with a searing account on how the present system of mathematics education is doing an outstanding job at destroying students' natural curiosity and interest in making patterns. Lockhart writes, "If I had to design a mechanism for the express purpose of destroying a child's natural curiosity and love of pattern-making, I couldn't possibly do as good a job as is currently being done in contemporary mathematics education."[9] In particular, Lockhart argues that the way in which students are being taught math is giving them a false conception of what mathematics is and how it is useful to us. Lockhart is strongly opposed to mandatory testing and he argues that teachers should not have to limit their instruction to simply meeting a set of curriculum standards. Lockhart asserts that "there is surely no more reliable way to kill enthusiasm and interest in a subject than to make it a mandatory part of the school curriculum."[10] However, if we don't make math a mandatory part of the school curriculum, how are schools supposed to be held accountable for teaching math to all students? While Lockhart's lament undoubtedly brings up several key points regarding why teachers need to change how they are teaching mathematics to their students, it also raises several questions concerning the practicality of his arguments.

Keith Devlin responds to some of these questions in "Lockhart's Lament—The Sequel." Devlin begins his response by stating that while Lockhart brings up many excellent points about how mathematics should be taught, the implementation of such ideas is just not realistic.[11] Lockhart laments the fact that teachers are required to follow a curriculum in their classroom because he believes that it limits their ability to teach mathematics in a creative and engaging way that will cultivate an appreciation for it in the minds of students. Devlin responds by saying that curricula are a necessary component of mathematics instruction. Devlin argues that since not every math teacher is well-qualified to teach math, they will not all be able to teach math without having a curriculum to follow. However, Devlin also argues that a thorough curriculum should not limit a teacher's ability to teach beyond the text.[12] Furthermore, Devlin insists that while developing a love of mathematics in students is a nice idea, it is realistically unnecessary. Devlin asserts that "industry needs few employees who understand what a derivative or an integral are, but it needs many people who can solve a differential equation."[13] Thus, Devlin concludes that while Lockhart's ideas may sound ideal, they are not feasible enough to transform our present-day mathematics education system. Nonetheless, both Devlin and Lockhart bring up valid points on how curricula should guide, but not limit, mathematics instruction in the classroom.

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[8] Ibid. (p. 4).
[9] Lockhart, Paul (2002). *A Mathematician's Lament* (p. 2). New York City: Bellevue Literary Press.
[10] Ibid. (p. 8).
[11] Devlin, Keith (2008). "Lockhart's Lament – The Sequel." *Devlin's Angle* (p. 2). Washington D.C.: Mathematical Association of America.
[12] Ibid.
[13] Ibid. (p. 3).

Inquiry-Based Learning as Effective

In his article titled "Linking Research and Teaching: Exploring Disciplinary Spaces and the Role of Inquiry-Based Learning," Mike Healey argues that inquiry-based learning provides students with a better understanding of the concepts that they are being taught. Healey argues that by being actively involved in the research of a particular discipline, students will have a stronger understanding of the foundational concepts for that discipline. Inquiry-based learning is a teaching strategy that promotes active learning and gets students involved in research.[14] What Healey means by "research" in this article is the construction of knowledge in a specific discipline; and he believes that it is important for students to be taught how different types of knowledge are assembled depending on the discipline. Healey notes that there are three dimensions to the design of every curriculum. The first dimension refers to the emphasis of the curriculum is on research content or on research processes and problems. The second dimension focuses on whether the students are treated as the audience or as the participants in the curriculum. The third dimension is on whether the teaching in the curriculum is teacher-focused or student-focused. In terms of these dimensions, Healey asserts that inquiry-based learning curricula emphasize research processes and problems, treat students as participants, and focus on the student.[15] According to Healey, student-focused approaches in curricula are focused on having students be active participants in class by guiding them in constructing their own knowledge.[16] However, it is important to realize that there is a difference between how a curriculum is written and how the teacher decides to use it as a tool for presenting the content because the teacher could still use a traditional style of curriculum to design a lesson that is inquiry-based and promotes active student engagement in class.

Healey describes inquiry-based learning as a "form of learning that is driven by a process of inquiry."[17] In his critique of direct instruction, Healey asserts that teaching is about more than simply transmitting information that is already known. Furthermore, he argues that direct instruction is only geared towards meeting the needs of the "most able students."[18] In his argument for linking research and teaching, Healey acknowledges that many teachers hold the belief that students need to understand certain concepts and be able to perform certain procedures depending on the discipline before they will be able to contribute anything to the research of that discipline. Thus, students are not being given the opportunity to participate in their academic community until much later in their education. This reality needs to change, and Healey believes that inquiry-based learning is the way in which to bring about this change. Healey argues that students get to engage in a vast range of diverse experiences in classrooms that utilize inquiry-based learning curricula, and he writes that "research-based learning structured around inquiry is one of the most effective ways for students to benefit from the research that occurs in a specific discipline."[19] While Healey observes that research and teaching are typically not linked in classrooms, he ultimately argues that for the purposes of pedagogical variety and student growth towards independence in learning, research and teaching need to be linked in the classroom.[20]

[14] Healey (2005). "Linking Research and Teaching" (p. 67).
[15] Ibid. (p. 69).
[16] Ibid. (p. 70).
[17] Ibid. (p. 73).
[18] Ibid.
[19] Ibid. (p. 74).
[20] Ibid. (p. 75).

Thus, Healey provides us with an argument as to why inquiry-based learning should be implemented in all curricula.

Inquiry-Based Learning as Ineffective

In contrast to Healey's perspective on inquiry-based learning, Paul Kirschner provides us with a different perspective on inquiry-based learning. In his article "Why Minimal Guidance During Instruction Does Not Work: An Analysis of the Failure of Constructivist, Discovery, Problem-Based, Experiential, and Inquiry-Based Teaching," Kirschner argues that inquiry-based learning is ineffective. He grounds his argument against inquiry-based learning in research on human cognitive architecture that has consistently shown that instruction with a minimal amount of guidance is less effective (i.e. students do not walk away from the lesson with a clear understanding of the important concepts) and less efficient (i.e. students do not learn the material as quickly) than direct instruction. He defines direct instruction as instruction that provides complete explanations of concepts and procedures to students, and he defines inquiry-based learning as a teaching strategy that requires students to construct concepts and procedures for themselves.[21] However, Kirschner asserts that students should not be expected to construct the important concepts and procedures of a subject for themselves.

Kirschner discusses the structure that makes up human cognitive architecture in order to illustrate the ineffectiveness and inefficiency of inquiry-based learning. He begins this discussion by defining learning in terms of long-term memory. Kirschner states that learning can be described as a "change in long-term memory."[22] According to Kirschner, working memory plays an important role in learning because it is the "cognitive structure in which conscious processing occurs."[23] However, a student's working memory has a very limited capacity for processing new information that has not yet been stored in the student's long-term memory.[24] Inquiry-based learning heavily relies upon students' working memories because it is designed to have students search for and discover concepts and procedures. Thus, Kirschner argues that since inquiry-based learning does not fit with what we know about human cognitive architecture, it is an ineffective teaching strategy.

Furthermore, Kirschner includes evidence on inquiry-based learning that supports his claim of it being an ineffective and inefficient teaching strategy. Kirschner cites several controlled experiments in which the conclusion was that students should be instructed directly instead of indirectly. In particular, Kirschner discusses a study that found inquiry-based learning to be successful if and only if students were engaged in direct instruction experiences that conveyed foundational knowledge before they began the learning that was driven by inquiry.[25] According to Kirschner, other studies on this topic have shown that inquiry-based learning typically results in students having more misconceptions and an incomplete conceptual understanding of a subject.[26] Based on evidence from various studies of human cognitive architecture in relation to inquiry-based learning, Kirschner provides us with a critical argument

[21] Kirschner, Paul A., John Sweller, and Richard E. Clark (2006). "Why Minimal Guidance During Instruction Does Not Work: An Analysis of the Failure of Constructivist, Discovery, Problem-Based, Experiential, and Inquiry-Based Teaching." *Educational Psychologist* (p. 76). London: Routledge.
[22] Ibid.
[23] Ibid. (p. 77).
[24] Ibid.
[25] Ibid. (p. 82).
[26] Ibid. (p. 84).

as to why curricula should focus solely on direct instruction in order to avoid what he believes to be the ineffectiveness and inefficiency of inquiry-based learning.

Differentiating Math Instruction

In this next section, I will focus on identifying various issues with teaching strategies in mathematics classrooms. Furthermore, how these issues are ineffectively and inefficiently meeting the different learning needs of students. In her article titled "Why and How to Differentiate Math Instruction," Amy Lin discusses the necessity of differentiating teaching strategies in mathematics classrooms. One major issue that Lin examines in her article is the lack of equity in terms of diverse learning needs in mathematics classrooms. Lin argues that since every student learns mathematics differently, teachers need to account for these learning differences in their instruction. In particular, Lin notes that not only does every student have different needs when it comes to learning, but they also have varying levels of mathematical ability based on their previous years of instruction. Lin argues that this difference between students' mathematical knowledge is a particularly challenging issue for teachers of grades 6-12.[27]

In order to account for these differences in mathematical ability and learning needs, Lin asserts that teachers need to incorporate big ideas, prior assessment, and choice into their lessons. She observes that many teachers feel limited by curriculum requirements, and so they focus their instruction on equipping students to meet narrow learning goals.[28] However, she argues that it is impossible to differentiate instruction that is formulated around too narrow of an idea.[29] Thus, Lin believes that big ideas are essential to effectively differentiating mathematics instruction because they form the framework for getting students to think about the fundamental principles of mathematics.[30] Moreover, Lin argues that prior assessment is a necessary component of effective math instruction because it provides teachers with important information regarding what their students need from the instruction that they receive. She also advocates for providing students with some element of choice either in how they learn a particular mathematical concept or in the follow-up activity for that lesson.[31]

Overall, Lin states that in order to differentiate math instruction efficiently, "teachers need manageable strategies that meet the needs of most of their students at the same time."[32] One suggestion for a manageable strategy that Lin gives in her article is asking open questions during instruction. Open questions are inclusive questions that are designed for a differentiation in responses based on each student's understanding.[33] These types of questions allow students of all mathematical levels to participate, and they help to correct the common misperception that many students have of mathematics being black or white.[34] Other strategies that Lin suggests are developing differentiated tasks around the same big idea and creatively incorporating student voice in the lesson. Essentially, this article provides teachers with strategies for differentiating

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[27] Lin, Amy, and Marian Small (2010). "Why and How to Differentiate Math Instruction." *More Good Questions: Great Ways to Differentiate Secondary Mathematics Instruction* (p. 2). New York City: Teachers College Press.
[28] Ibid. (p. 4).
[29] Ibid.
[30] Ibid.
[31] Ibid. (p. 6).
[32] Ibid. (p. 7).
[33] Ibid. (p. 10).
[34] Ibid.

mathematics instruction and it advocates for the importance of differentiating how students are being taught.

Learning Needs in Mathematics Classrooms

Similar to Lin's argument, Mazlini Adnan, in "Learning Style and Mathematics Achievement among High Performance School Students," argues that teachers need to use a variety of teaching strategies in order to meet the different learning needs of students. Adnan argues that some students learn better from direct instruction while other students prefer inquiry-based learning. According to Adnan, this preference for either direct instruction or inquiry-based learning is dependent on how each student processes information.[35] He conducted a study to determine whether or not there was a correlation between learning styles and high performance in mathematics. His results showed that "the relationship between active learning styles and mathematics achievement is very weak."[36] Inquiry-based learning incorporates a high level of active learning in the classroom, which naturally benefits students who have active learning styles. However, since the students with active learning styles typically had lower mathematics achievement than their peers in Adnan's study, it would seem that active learning is not being given a prominent position in math classrooms. Thus, Adnan's study illustrates how mathematics instruction is not meeting the learning needs of all students.

The work of Roza Leikin in "Exploring Mathematics Teacher Knowledge to Explain the Gap Between Theory-Based Recommendations and School Practice in the Use of Connecting Tasks" further explores the issue that Adnan identifies with teaching strategies in mathematics classrooms. In particular, Leikin analyzes why "teachers find it difficult to teach multiple solution strategies to problems."[37] Leikin argues that it is very important for mathematics teachers to intentionally provide opportunities in their classrooms for students to solve problems in different ways because it will help to develop their students' conceptual understanding of mathematical principles. The solution to this issue of math teachers not teaching multiple solutions to problems that Leikin provides us with in her article is centered on the idea of incorporating multiple-solution connecting tasks into mathematics instruction. Leikin defines a multiple-solution connecting task as a task that combines different mathematical concepts in such a way that it can be solved in multiple ways.[38] Essentially, these are tasks that can be completed using different procedures and strategies and allow for a divergence in how students think about and approach them. Leikin asserts that mathematics instruction in the United States does not utilize multiple-solution connecting tasks. However, Leikin cites studies that show while multiple-solution connecting tasks are not part of mathematics instruction in the United States, Germany, and Israel, they are part of mathematics instruction in China and Japan.[39]

Despite the research that supports implementing multiple-solution connecting tasks into mathematics curricula, Leikin observes that this implementation is not actually happening in the classroom. Leikin argues that this gap between theory and practice in teaching strategies is a

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[35] Adnan, Mazlini (2013). "Learning Style and Mathematics Achievement among High Performance School Students." *World Applied Sciences Journal* (p. 392). Malaysia: IDOSI Publications.
[36] Ibid. (p. 396).
[37]Leikin, Roza, and Anat Levav-Waynberg (2007). "Exploring Mathematics Teacher Knowledge to Explain the Gap Between Theory-Based Recommendations and School Practice in the Use of Connecting Tasks." *Educational Studies in Mathematics* (p. 349). Germany: Springer Science + Business Media B.V.
[38] Ibid. (p. 350).
[39] Ibid. (p. 351).

result of teachers' focus in the classroom being mainly on meeting curriculum standards.[40] In particular, Leikin asserts that teachers consider multiple-solution connecting tasks to be an "insecure environment" because students may get confused by the existence of more than one solution to a problem.[41] Leikin argues that the problems and tasks that teachers assign to their students are result orientated in regards to the curriculum standards.[42] Thus, Leikin concludes her article by advocating for a change in mathematical curriculum and testing in order to cultivate a classroom setting in which multiple-solution connecting tasks could be reasonably incorporated into the instruction.

Connection to this Project

The effectiveness and efficiency of direct instruction versus inquiry-based learning in high school math classrooms is an ongoing discussion and topic of research and there are valid arguments that support both sides of the debate. The purpose of my project is not to advocate for either direct instruction or inquiry-based learning; rather, its purpose is to provide a descriptive curricula analysis of these contrasting approaches to teaching mathematics and design a five-day lesson plan that employs the strengths of both approaches. The five-day lesson plan that I wrote for this project takes into account the findings from the various studies on effective teaching strategies for mathematics that were discussed in this literature review. While no lesson plan is flawless, the lesson plan for this project is intended to provide an example for how math instruction can be differentiated in a way that includes elements of both a direct instruction and an inquiry-based learning approach to teaching. In summary, this literature review has explored different issues and perspectives related to how mathematics is being taught in order to construct the setting in which my analysis and synthesis of direct instruction and inquiry-based learning curricula will take place.

[40] Ibid. (p. 366).
[41] Ibid.
[42] Ibid.

Methodology

The framework that I developed for my analysis of the direct instruction and inquiry-based learning curricula is focused on examining how well each curriculum meets the three expectations for rigor in math education that are presented in the Common Core State Standards. As mentioned previously in the introduction, experts believe that mathematics education in the United States has failed to provide students with a strong foundation from which future mathematical knowledge can be built.[43] One reason for this failure is that curricula are focused on covering a broad expanse of topics, which results in very little time spent going in depth on any particular mathematical topic.[44] In an attempt to resolve this issue, the Common Core State Standards were established with the goal of implementing a deeper and more rigorous curriculum in mathematics education, and the ways in which the concept of rigor has been designed and developed in these standards are among the main reasons why they have become so influential in our current education system. Thus, it seems particularly relevant to frame my curriculum analysis around the idea of rigor in mathematics education.

As defined by the authors of the Common Core State Standards, rigor is the pursuit of conceptual understanding, procedural skill and fluency, and applications in mathematics with equal intensity.[45] Conceptual understanding, while difficult to define since math education researchers have not yet come to an agreement on its definition, refers to students' understanding of concepts and how they relate to each other.[46] It can be demonstrated by classroom discussions about the mathematical reasoning behind an answer to a particular math problem, simple computational problems that link the solution to a conceptual question, making connections between functions and graphs, generating examples of a concept, using key vocabulary words in problems, and assigning problems that construct a variety of quantitative relationships. In my analysis of the direct instruction and inquiry-based learning curricula, the curricula are evaluated on how well they develop conceptual understanding in their lessons on trigonometric functions. My analysis examines what aspects of each curriculum are concentrated on developing students' conceptual understanding and what aspects of each curriculum seem to be lacking in this area.

In order to construct a framework for how the different elements of each curriculum are or are not developing students' conceptual understanding in my analysis, I utilized the level of cognitive demands scale in mathematics classrooms that Margaret Schwan Smith and Mary Kay Stein developed in *Selecting and Creating Mathematics Tasks: From Research to Practice*. Smith and Stein referred to lower-level demands as memorization and procedures without connections.[47] Lower-level demands do not require students to cognitively engage with the mathematical concepts that are being taught in the lesson. The tasks that are attributed as having lower-level demands often involve simply using memorization or following a procedure that was shown in class to produce the correct answer. Smith and Stein defined higher-level demands as procedures with connections and doing mathematics.[48] Essentially, higher-level demands lead to

[43] *High School Publishers' Criteria for the Common Core State Standards for Mathematics* (p. 2).
[44] Ibid.
[45] Ibid. (p. 3).
[46] Ibid. (p. 9).
[47] Smith, Margaret Schwan, and Mary Kay Stein (1998). "Selecting and Creating Mathematics Tasks: From Research to Practice." *Mathematics Teaching in the Middle School* (p. 348). Pittsburgh: National Council of Teachers of Mathematics.
[48] Ibid.

the development of students' conceptual understanding by requiring students to develop their own procedure for how to solve mathematical problems and by requiring them to make connections between key mathematical relationships. This scale on the level of cognitive demands in mathematics classrooms is used in my curricula analysis to establish a clear framework for how each curriculum develops students' conceptual understanding of trigonometric functions.

The second expectation for rigor in mathematics education given by the Common Core State Standards is procedural skill and fluency. The goal of procedural skill and fluency is to equip students with the strategies and practices necessary to make them fluent in mathematical skills. Procedural fluency refers to a student's "knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently."[49] While developing students' procedural skill and fluency allows them lots of practice with solving computational and procedural problems, it is also connected to the development of their conceptual understanding of how the procedures work algebraically. Since these two components of rigor are so tightly interwoven, it may be difficult at times to tease apart when it is conceptual understanding and when it is procedural fluency. In my curricula analysis, I evaluated how each curriculum incorporated procedural skill and fluency into their lessons on trigonometry by analyzing how each curriculum engaged students in working with problems involving trigonometric functions. Furthermore, how each curriculum engages students in working with these problems can be categorized as procedures with connections or as procedures without connections, depending on how much explanation of the procedure is requested. Procedures with connections means that students must provide a rationale for how they solved a problem or develop their own procedure for solving a problem. Procedures without connections means that students can simply follow a procedure that has been shown to them without the exploring the why behind how it works.

The third expectation for rigor is engaging students in applications of mathematical concepts. One way in which curricula incorporate applications into their lesson plans is by writing problems in a real-world context that are designed for students to work through in either a collaborative or an independent setting. These problems typically involve making practical assumptions based on the context of the problem, developing a procedure to solve the problem, and making connections between mathematical concepts. In my analysis, I assessed the connections that the direct instruction and inquiry-based learning curricula made between trigonometric functions and real-world applications.

Conceptual understanding, procedural skill and fluency, and applications are seen in conjunction as well as in disjunction with each other in mathematics curricula.[50] Some learning tasks just focus on developing and addressing one component of rigor while other learning tasks interweave two or more of the components. The Common Core State Standards sets the expectation that all three of the components of rigor in mathematics education are presented with equal intensity. While not every learning task or lesson may integrate these three components with equal intensity, every curriculum as a whole is expected to do so. My analysis of the direct instruction and inquiry-based learning curricula will investigate how each curriculum balances the three components of rigor in relation to each other. In summary, conceptual understanding,

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[49] Kilpatrick, Jeremy, Jane Swafford, and Bradford Findell (2001). *Adding It Up: Helping Children Learn Mathematics* (p. 121). National Research Council, Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

[50] *High School Publishers' Criteria for the Common Core State Standards for Mathematics* (p. 10).

procedural skill and fluency, and applications are the three expectations for rigor in mathematics education, and they establish the framework for my analysis of the direct instruction and inquiry-based learning curricula.

Findings

I conducted an analysis of two direct instruction and two inquiry-based learning curricula. Curriculum A[51] and Curriculum B[52] are the direct instruction curricula and Curriculum C[53] and Curriculum D[54] are the inquiry-based learning curricula.

In order to numerically assess how the three components of rigor are integrated into each curriculum, I classified every quiz and test question in the trigonometry unit of each text into one of the seven categories shown in Figure 1 (page 15). The language that the assessment questions used was crucial in how I determined which category each question fell under. Questions that used verbs like solve, evaluate, and simplify were placed in the procedural skill and fluency category. If the question also asked for a graph, required the student to know key vocabulary words (such as amplitude, terminal side of an angle, or unit circle), or asked the student to provide an explanation of their work, then the question was classified as conceptual understanding + procedural skill and fluency. If the question asked students to solve a problem embedded in a real-world scenario, it was classified as conceptual understanding + procedural skill and fluency + applications because students had to use their conceptual knowledge to apply their procedural skills to a new situation. Quiz and test questions were classified as conceptual understanding if they asked students to think critically, define a key term, or make connections between concepts without requiring any procedural work. In Curriculum C, some questions were categorized as conceptual understanding + applications because they asked students to think critically and make connections between a key concept and a real-world scenario without asking for any procedural work. None of the questions were classified as either applications or procedural skill and fluency + applications because all of the problems that contained a real-world application also had conceptual understanding interwoven into the fabric of the problem.

In Curriculum B, every assessment question had a procedural skill and fluency focus to it, but only 42% of the questions contained an element of conceptual understanding and only 8% had a real-world application embedded in the problem. In over 50% of the problems, procedural skill and fluency was the only component of rigor that was being evaluated. The questions that assessed conceptual understanding alongside procedural skill and fluency expected students to know important vocabulary terms and to be able to graph trigonometric functions. Curriculum A also had a significant emphasis on procedural skill and fluency over the other two components of rigor, for 99% of its problems had an element of procedural skill and fluency, while only 53% and 4% of the problems had an element of conceptual understanding and applications, respectively. The most noticeable difference between the assessment compositions of the two direct instruction curricula is related to the findings for conceptual understanding + procedural skill and fluency. Curriculum A pairs procedural skill and fluency with conceptual understanding in 48% of its assessment problems, while Curriculum B makes that same pairing in only 34% of its assessment problems. In several test questions in Curriculum A, students were

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[51] Benson, John, Sara Dodge, Walter Dodge, Charles Hamberg, George Milauskas, and Richard Rukin (1991). *Teacher's Edition Algebra 2 and Trigonometry* (pp. 610-639). Illinois: McDougal, Littell & Company.

[52] Larson, Ron, Laurie Boswell, Timothy D. Kanold, and Lee Stiff (2012). *Algebra 2* (pp. 610-677). Florida: Houghton Mifflin Harcourt Publishing Company.

[53] Hirsch, Christian R., James T. Fey, Eric W. Hart, Harold L. Schoen, and Ann E. Watkins (2008). *Core-Plus Mathematics (Course 2): Contemporary Mathematics in Context* (pp. 457-487). New York City: McGraw Hill.

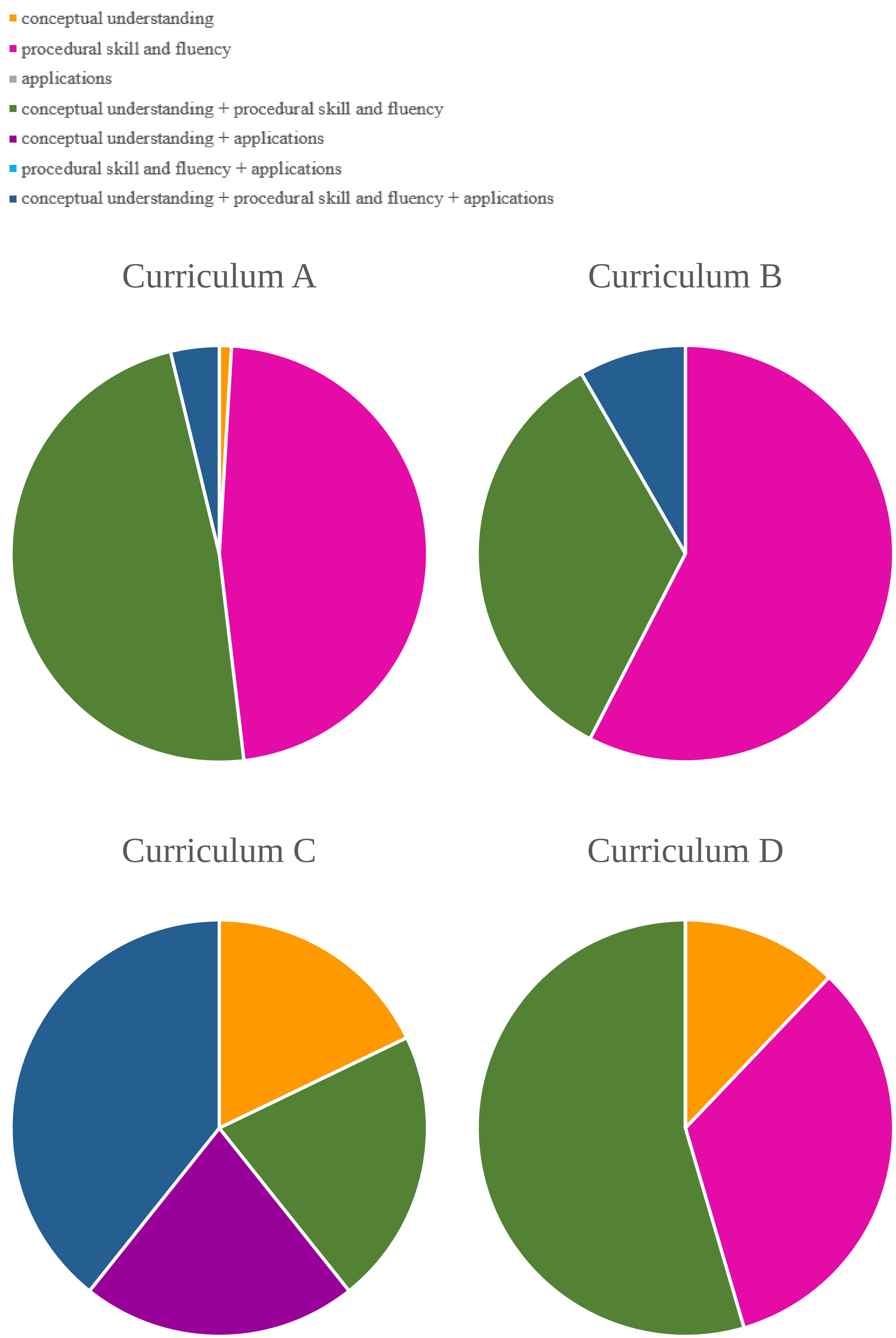
[54] Cuoco, Al (2009). *Algebra 2: Center for Mathematics Education Project* (pp. 680-777). New Jersey: Pearson Education, Inc.

assessed on proving trigonometric identities, analyzing or manipulating a graph, and explaining their answer, all of which pair conceptual understanding with procedural skill and fluency. In both of the direct instruction curricula, assessments are primarily composed of questions that evaluate procedural skill and fluency, and while real-world applications are present in the quizzes and tests, they are by no means a main focus.

One major similarity between Curricula A, B, and D is that they all prioritize procedural skill and fluency in their assessments. Approximately 88% of Curriculum D's assessment problems had procedural skill and fluency in them, and 33% of them were solely focused on procedural skill and fluency. The majority of these problems were asking students to solve equations and simplify expressions involving trigonometric functions. However, Curriculum D places a greater emphasis on conceptual understanding than either of the direct instruction curricula because 67% of its assessment questions draw on students' conceptual understanding, while only 53% and 42% of the assessment questions in Curricula A and B, respectively, do the same.

As we can see in Figure 1, Curriculum D's assessment composition looks fairly similar to those of the direct instruction curricula, but Curriculum C's assessment composition looks significantly different. In particular, Curriculum C includes a real-world application of some sort in more than 61% of its assessment problems for the trigonometry unit, which is a much greater percentage than the other three curricula (4%, 8%, and 0% of assessment problems have a real-world application in Curricula A, B, and D, respectively). Curriculum D is on the other end of the spectrum for integrating real-world applications into its content, for it does not have any assessment questions that contain applications in its unit on trigonometry, which is even more extreme than the direct instruction curricula since they both had at least a few problems that tied in real-world applications. Another significant way in which Curriculum C's assessment questions differ from the direct instruction curricula is that it prioritizes measuring students' conceptual understanding over their procedural skill and fluency. All of its assessment problems have conceptual understanding interwoven into them, while only 61% contain a procedural skill and fluency component, which is significantly less than what we see in the other three curricula. All in all, the two direct instruction curricula have very similar assessment compositions; however, that is not the case when it comes to the inquiry-based learning curricula, for Curriculum C and Curriculum D illustrate two drastically different designs for using an inquiry-based approach to teaching trigonometry.

Figure I. Assessment Composition



Direct Instruction Analysis

I focused my analysis on how each direct instruction curriculum integrates conceptual understanding, procedural skill and fluency, and applications into its instruction on trigonometry. In particular, I analyzed where these three components of rigor, as defined by the Common Core State Standards, appear in the structure of each curriculum, and I analyzed how they are developed throughout each curriculum's direct instruction approach to teaching.

Curriculum A

The first direct instruction curriculum that I analyzed is titled *Teacher's Edition Algebra 2 and Trigonometry*.[55] There are three main parts to each lesson in Curriculum A: the first part is an introduction to the mathematical topic that will be the focus of the lesson; the second part consists of sample problems and solutions; and the third part includes warm-up exercises in addition to a problem-set. The introduction directly states and explains the mathematical content for the lesson in brief and concise sections that are then followed by a couple of examples to illustrate particular concepts. Typically, these examples include a graph or diagram of the situation that is being described in the problem. In this part of the curriculum's structure, the lower-level cognitive demand of memorization is present because the text has the important concepts and definitions related to trigonometric functions written in boldface font, with the intent that students will come away from the lesson with these ideas rooted in their minds. For example, students are expected to know the definitions of the six trigonometric functions [$\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$, $\csc(\theta)$, $\sec(\theta)$ and $\cot(\theta)$], the Pythagorean identities, and how $\cos(\theta)$ and $\sin(\theta)$ relate to a point on the unit circle.[56] The lower-level demand of memorizing these important concepts and definitions is serving as a foundation for continued learning in this lesson by requiring students to become familiar with these concepts and definitions so that procedures with connections can be introduced later on in the lesson. Moreover, this lower-level demand of memorization, which establishes a foundation for the lesson, is furthered into higher-level cognitive demands through the procedural skill and fluency aspect of rigor. In the second and third parts of its structure, this curriculum works through a plethora of sample problems and individual exercises that require students to become familiar with the key definitions and concepts related to trigonometric functions.

One way in which Curriculum A incorporates conceptual understanding into its lesson plans on trigonometric functions is by asking students to connect what they have learned about in previous lessons to the new lesson topic. In each of the lessons in this curriculum, there is a section on the side of the textbook that is titled "Communicating Mathematics." This section in the lesson on trigonometric functions asks students to write a short paragraph with diagrams that illustrate how the Pythagorean Theorem is used to construct the Pythagorean identities that they are supposed to memorize.[57] Students learned about the Pythagorean Theorem previously in the textbook, and now they are being asked to connect it to what they are currently learning about trigonometric functions. This connection between what students are supposed to memorize and the development of their conceptual understanding shows how the lower-level cognitive demand

[55] Benson, John, Sara Dodge, Walter Dodge, Charles Hamberg, George Milauskas, and Richard Rukin (1991). *Teacher's Edition Algebra 2 and Trigonometry* (pp. 610-639). Illinois: McDougal, Littell & Company.
[56] Ibid. (pp. 610-612).
[57] Ibid. (p. 612).

of memorization can be transformed into a higher-level cognitive demand by requiring students to think about how two mathematical concepts are interrelated.

Additionally, each lesson in this curriculum has a section on the side of the textbook that is titled “Cooperative Learning.” In terms of rigor, this section is designed to further students’ conceptual understanding of key ideas that are needed to form a strong foundation for future knowledge in a particular content area. There is also a section on the side of the textbook that alerts teachers to conceptual stumbling blocks their students may face in understanding important mathematical concepts. In order to identify whether or not their students have any misconceptions about the content that has just been presented to them, the teachers have a short section titled “Checkpoint” on the side of their textbook immediately following the introduction of the mathematical content. In the trigonometric functions section of this curriculum, the problems that are listed in the checkpoint section are solely focused on the conceptual understanding and the procedural skill and fluency aspects of rigor, and they do not contain any applications.

Nonetheless, the three components of rigor are interwoven in a small proportion of the sample problems and solutions that make up the second part of Curriculum A’s structure. For example, one of the sample problems uses the real-world application of constructing the roof of a house by asking students to find the angle that the roof makes with the horizontal.[58] This problem requires students to use what they know about trigonometric functions to strategically develop a procedure that will lead them to the correct answer. Since students have to develop their own procedure, this problem requires a higher-level cognitive demand. Thus, conceptual understanding, procedural skill and fluency, and applications are all present in this sample problem.

However, while there is one sample problem that includes an application, there are five other sample problems that do not contain references to any real-world applications, and in the lesson on trigonometric functions, only one out of forty problems contain a real-world application. This disproportion is also seen in the warm-up exercises and problem-set that make up the third part of this curriculum’s structure. Instead of interweaving all three of the components of rigor, these problems are primarily focused on the procedural skill and fluency aspect of rigor. For example, the problems in this section are asking students to find values, expressions, and angle measures using trigonometric functions, which means that these problems are mainly focused on developing students’ procedural skill and fluency in trigonometry.

Moreover, the text includes a list of problem-set notes and strategies for teachers to reference as their students work through the questions in the problem-set. For example, one of the questions requires students to make a connection between the tangent function and the x-and-y-values of a coordinate point. The strategy note to the teacher warns that this connection may take some time for students to make, and that the students will need to identify how the Pythagorean Theorem relates to the question in order to find the answer.[59] In this problem, students are required to use higher-level cognitive demands in order to make these connections. Therefore, conceptual understanding is being integrated alongside procedural skill and fluency in the problem-set for trigonometric functions.

Throughout the three different parts of Curriculum A’s structure, the three components of rigor are seen in conjunction as well as in disjunction with each other. However, it is apparent that conceptual understanding, procedural skill and fluency, and applications are not being

[58] Ibid. (p. 613).
[59] Ibid. (p. 623).

integrated with equal intensity into this direct instruction curriculum. Ultimately, we can conclude for the trigonometry unit of the text that while rigor is integrated into Curriculum A, it is not yet meeting the expectations of the Common Core State Standards for rigor in regards to its disproportionately greater focus on procedural skill and fluency and how it relates to conceptual understanding than on real-world applications.

Curriculum B

The second direct instruction curriculum that I analyzed is titled *Algebra 2* and it has two chapters in its unit on trigonometry.[60] The first chapter is titled "Trigonometric Ratios and Functions" and the second chapter is titled "Trigonometric Graphs, Identities, and Equations."[61] Each chapter begins with a section on prerequisite skills that contains a brief problem set on previously learned material. This section checks students' understanding of key vocabulary terms and algebraic skills that they will need to be able to utlize in the new chapter. At the start of every chapter as well as at the start of every lesson, Curriculum B introduces a new topic in a "Before, Now, and Why" format by connecting it to what students had been learning before, what students will be learning now, and why they will be learning it. The text seeks to make connections for students by introducing what they will now be learning in the context of what they have learned previously. By explicitly making this connection for students, Curriculum B helps to further students' conceptual understanding of how different topics in mathematics are related, which helps students to see everything that they are learning as interwoven and interconnected instead of as discrete and disconnected. However, since the text is making the connections for the students instead of leading the students to make the connections for themselves, it could lessen the depth and impression that these connections make on the students' understanding and insight.

Furthermore, the "Before, Now and Why" section answers the commonly heard refrain in mathematics classrooms of "why are we learning this?" Answering this question not only provides students with a better understanding of how a seemingly abstract concept can actually be useful, but it also orients the students in the direction that the lesson will be taking them. Curriculum B employs the "Why" strategy in its introduction to trigonometry by explaining that trigonometry can be used to "find lengths and areas in real life."[62] The real life example that the text illustrates is finding the area of a step on a spiral staircase. As mentioned in the section on methodology, the three components of rigor are often seen in conjunction with each other, and the "Before, Now, and Why" section of Curriculum B is a great example of that relationship. The "Before" section connects previously learned procedures and skills to what students will be learning in the new lesson or chapter, which guides them in understanding how different concepts are related, and this understanding requires a higher-level cognitive demand from students. The "Why" section ties in applications to conceptual understanding by framing the concepts that students will be learning in a way that highlights how these concepts can be useful in real life.

The lessons in Curriculum B are written in the following pattern: a key concept is introduced, one or two examples are given for that key concept, a guided practice section with problems similar to the examples comes next; then, another key concept is introduced and the

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[60] Larson, Ron, Laurie Boswell, Timothy D. Kanold, and Lee Stiff (2012). *Algebra 2* (pp. 610-677). Florida: Houghton Mifflin Harcourt Publishing Company.
[61] Ibid. (p. xvi-xvii).
[62] Ibid. (p. 555).

pattern continues. All of the key concepts are directly explained and no investigative work is required of the students. These concepts are presented in a way that focuses on equipping students with the tools needed to do the problems and exercises in that section; thus, its focus is much more on procedural skill and fluency than on developing students' conceptual understanding. For example, when the text introduces students to evaluating trigonometric functions, it outlines a procedure with three steps that students can use whenever they are solving for an angle θ.[63] Even though the procedure is Curriculum B's main focus when it comes to the key concept, the curriculum also includes a diagram, which is placed to one side in the text, of the signs for sin(θ), cos(θ), tan(θ) in the four quadrants of a graph. This diagram serves to further develop students' conceptual understanding when it comes to evaluating trigonometric functions, but it should be noted that students could simply memorize (lower-level cognitive demand) the diagram without really having to investigate how the signs of the trigonometric functions are related to the quadrants of a graph. We could even go so far as to say that the text really isn't developing students' conceptual understanding at all since it doesn't explain *why* the trigonometric functions are positive or negative in each quadrant.

Following each lesson is an exercises section that contains approximately thirty to fifty problems. The exercises are split into two parts: skills practice and problem solving. The skills practice section is primarily based on procedural skill and fluency, but as we mentioned in the methodology section of this paper, procedural skill and fluency and conceptual understanding can be difficult to tease apart at times because conceptual understanding plays an important role in whether students understand how to approach and procedurally solve skills-based exercises. Meanwhile, the problem-solving exercises have all three components of rigor interwoven into every problem. In this section, every exercise contains a real-world scenario (applications), asks students to solve for a numerical value (procedural skill and fluency), and requires them to orient their answer in the context of the real-world scenario and explain their reasoning (conceptual understanding).

One notable feature of Curriculum B is a section titled "Problem Solving Workshop" that follows immediately after one of the lessons in every chapter. This section builds off a particular problem-solving method that students were taught in an example from the lesson by listing alternative methods for solving that same example. Understanding that there are multiple methods that can be used to solve the same problem is a very important part of a student's mathematical development, particularly in regard to procedural fluency. However, students are still not being challenged to find alternative methods on their own because the text is continuing to show them the different methods that can be used, so students are merely mimicking the methods as they work through the practice problems. While students would hopefully use a higher-level of cognitive reasoning when engaging with alternative methods for solving the same problem by trying to understand why these different methods will ultimately lead to the same answer, students could just follow the alternative methods that are shown without wrestling with the "why" behind each of them. However, if students could potentially follow the procedures without really having any knowledge of when or why it is appropriate to use them, should this problem-solving section of the curriculum even be categorized as procedural fluency?

There are two distinct features in each chapter of Curriculum B that deviate from what we would typically expect to see in a direct instruction curriculum. The first is a section titled "Mixed Review of Problem Solving" and it is present in every chapter and comprised of several multi-step problems that contain a real-world scenario. This section looks very similar to the

[63] Ibid. (p. 572).

kinds of problem-sets that we see in inquiry-based learning curricula. The second nontraditional feature that I found in each chapter in Curriculum B is an inquiry-based activity that has students explore a particular key concept. For example, in the first chapter on trigonometry, this inquiry-based activity has students explore the law of sines by drawing a triangle, measuring the angles and side lengths, calculating the ratios $\frac{\sin A}{a}$, $\frac{\sin B}{b}$, and $\frac{\sin C}{c}$, and drawing conclusions about their observations. This activity requires students to engage at a higher cognitive level with the mathematical content by making connections between what they were seeing with the ratios and what they had just learned about the law of sines. One feature that is important to note about this explorative activity is that it comes after a lesson on the law of sines, which means that students have already been explicitly taught that this law exists, so they are not actually discovering it for themselves. In a typical inquiry-based learning curriculum, we would expect students to do an activity similar to this one before being told about the law of sines.

Real-world applications are present in Curriculum B, but they are not a foundational part of the text. They are typically found in one example in each lesson, the problem-solving section of the post-lesson exercises, and the "Mixed Review of Problem Solving" section in each chapter. They are not present in the chapter summaries for the trigonometry unit, nor are they a focus in the chapter quizzes and tests. The main emphasis in Curriculum B is on procedural skill and fluency because its development is the main theme of the key concepts, examples, guided practice, exercises, and assessments. While conceptual understanding is often tied into the procedural skill and fluency components of this curriculum, how the text is actually measuring students' conceptual understanding is unclear because students could use lower-level cognitive thinking to memorize concepts and mimic procedures in place of higher-level cognitive thinking, which would require them to pursue a deeper understanding of how trigonometric concepts and procedures are connected. For the trigonometry unit of Curriculum B, we can conclude that while the three components of rigor are integrated into the text, it is not yet meeting the expectations of the Common Core State Standards because of its disproportionately greater focus on procedural skill and fluency, its limited connections to real-world applications, and its lack of a measurable outcome when it comes to evaluating students' conceptual understanding.

Inquiry-Based Learning Analysis

As with the direct instruction curricula, I focused my analysis on how each inquiry-based learning curriculum integrates conceptual understanding, procedural skill and fluency, and applications into its instruction on trigonometry. In particular, I analyzed where these three components of rigor, as defined by the Common Core State Standards, appear in the structure of each curriculum, and I analyzed how they are developed throughout each curriculum's inquiry-based approach to teaching.

Curriculum C

The first inquiry-based learning curriculum that I analyzed is titled *Core-Plus Mathematics (Course 2): Contemporary Mathematics in Context*.[64] The trigonometric functions unit of Curriculum C is structured into three sections that are referred to as investigations by the text. These three investigations all follow identical formats, and they make up almost the entirety of the lesson on trigonometric functions. The rest of the lesson consists of a brief introduction to trigonometric functions in a real-world context and it ends with an independent practice section.

The lesson on trigonometric functions in this inquiry-based learning curriculum begins with a section titled "Think About This Situation." This section brings in a real-world contextual problem that illustrates the applicability of the particular mathematical concept that is being explored in each lesson. For example, this curriculum uses a jack mechanism to show students how the measures of the sides and angles of a triangle are interconnected.[65] This section of the lesson asks the students to think about how the measures of the sides and angles of the triangle change in relation to each other as the rod of the jack mechanism is turned. In this introduction to the lesson, the text has students connect what they learned about triangles as rigid figures in a previous lesson to what they are going to learn about triangles in regards to trigonometric functions. This section requires a higher-level cognitive demand from students, but it does not require them to perform any mathematical calculations. Thus, this section of the inquiry-based learning curriculum includes a conjunction of conceptual understanding and applications, but it excludes the procedural skill and fluency component of rigor.

The first part of the investigations in this curriculum poses a couple of questions for students to focus on answering as they work through the problems and real-world scenarios that are given in the next part of the investigation. For example, the second investigation in this lesson is titled "Measuring Without Measuring," and it focuses the students' attention on the question of how trigonometric functions can be used to calculate distances that cannot be measured precisely.[66] The text does not directly provide the students with the information that they need to answer this question. Instead, the text instructs students to use what they learned about trigonometric functions in the previous investigation to answer this question. Essentially, this curriculum is guiding students on how to conceptually think about mathematical theories, but it is not instructing them on how to procedurally apply these concepts to make calculations.

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[64] Hirsch, Christian R., James T. Fey, Eric W. Hart, Harold L. Schoen, and Ann E. Watkins (2008). *Core-Plus Mathematics (Course 2): Contemporary Mathematics in Context* (pp. 457-487). New York City: McGraw Hill.
[65] Ibid. (p. 458).
[66] Ibid. (p. 467).

The second part of the investigations in this curriculum consists of integrating the three components of rigor into multi-step problems that build off of the questions that are posed in the first part of the investigations. The first part of each of these problems typically involves performing a simple mathematical calculation using trigonometric functions. In order for students to be able to make these simple computations, they need to have a conceptual understanding of how to use trigonometric functions, and they need the procedural skills to be able to correctly perform the actual computation. The mathematical concepts that students need to memorize are written in boldface font in this section of the investigation. However, since this inquiry-based learning curriculum does not directly teach students the procedures for how to use trigonometric functions in making mathematical calculations, students must do more than simply memorize the trigonometric functions because they are being required by the text to develop their own procedure for how to apply what they know about trigonometric functions to finding side and angle measures of triangles. Having students develop their own procedures is an important aspect of procedural skill and fluency since it is requiring students to use methods that make sense to them, even if they are not necessarily using the standard method. Thus, this curriculum requires students to employ a higher-level cognitive demand when they are solving these investigative problems.

In the second section of each investigation, the other parts of these problems are focused on a variety of real-world applications involving scenarios in which triangular diagrams can be constructed. For example, the height of a real-world structure is compared to the height of a person in one of the problems, and the problem asks students to determine some of the lengths and angles between the person and the structure using trigonometric functions.[67] Hence, these investigative problems consist of an integration of all three of the components of rigor.

Furthermore, the third part of the investigations in this curriculum includes a section titled "Summarize the Mathematics." Distinct from the previous aspects of these investigations, this section does not include applications, and it is primarily focused on developing students' conceptual understanding. In the third investigation on trigonometric functions, this section asks students how they could find particular side and angle measurements of a triangle based on what pieces of information are given to them.[68] However, this section does not ask students to actually calculate those measurements, which reveals a certain lack of intensity in this curriculum in regards to the procedural skill and fluency component of rigor. While asking students to think about how they could build their own procedures for finding angle measures and side lengths certainly relates to their development of procedural skill and fluency, the absence in this curriculum of having students actually make those mathematical calculations using an efficient method is undeniably concerning.

The final part of the investigations in this curriculum is titled "Check Your Understanding" and it smoothly integrates all three components of rigor into one real-world contextual problem that involves developing and following a procedure to determine distances and angle measures. Following the three investigations in the lesson on trigonometric functions in this curriculum, there is a section titled "On Your Own" that includes problems specifically for applications, connections, reflections, extensions, and review. The three components of rigor are presented with varying levels of intensity in these different sets of problems. For example, the review problems are connected to the procedural skill and fluency component of rigor, the

[67] Ibid. (p. 468).
[68] Ibid. (p. 473).

reflection problems are connected to the conceptual understanding component of rigor, and the extension problems are an integration of all three of the components of rigor.

In conclusion, the three components of rigor are all present throughout the structure of this inquiry-based learning curriculum, but they are not all presented with equal intensity. The Common Core State Standards set the expectation for rigor as the pursuit, with equal intensity, of conceptual understanding, procedural skill and fluency, and applications. There is a stronger emphasis on conceptual understanding and how it relates to real-world applications than on procedural skill and fluency. Since this inquiry-based method of teaching does not include direct instruction on mathematical procedures involving trigonometric functions, students are guided by the text into developing their own procedures. This method of teaching results in a greater focus on conceptually developing mathematical procedures instead of practicing them in order to gain procedural fluency. <u>In essence, we can conclude for the trigonometry unit of the text that while rigor is interwoven into this inquiry-based learning curriculum, it is not yet meeting the expectations of the Common Core State Standards for rigor in regards to its disproportionately greater concentration on conceptual understanding and how it relates to real-world applications than on procedural skill and fluency.</u>

Curriculum D

The second inquiry-based learning curriculum that I analyzed is titled *Algebra 2: Center for Mathematics Education Project*.[69] Curriculum D's final chapter is an introduction to trigonometry. It begins with a "Chapter Opener" that seeks to activate students' prior knowledge by reviewing key ideas on right triangles, namely, similarity and the AA Theorem. Then, the text connects these concepts to the sine, cosine, and tangent ratios in right triangles by directly stating them as definitions, which is a different approach than what we saw in our analysis of the other inquiry-based learning curriculum. In Curriculum C, students discovered the trigonometric ratios for themselves through a series of investigative problems working with right triangles and side ratios before the text provided them with definitions of sine, cosine, and tangent. However, in Curriculum D, the text immediately gives definitions for the trigonometric functions and then has students do investigative work using those functions as their primary tools.

Curriculum D's chapter on trigonometry is divided into three investigations: Trigonometric Functions, Graphs of Trigonometric Functions, and Applications to Triangles. Each investigation is divided into three to five important subtopics that relate to the main topic. Curriculum D begins each investigation by telling students what they will be able to do by the end of the investigation, and it has this section broken down into three main categories of sentence stems that, interestingly enough, correspond with the three components of rigor. The first sentence stem is "You will be able to answer questions like these…" and it is set up to measure students' conceptual understanding by posing questions like "How can you extend the definitions of sine, cosine, and tangent to any angle, not just acute angles?" and "What is the relationship between the equation of the unit circle and the Pythagorean Identity?"[70] The second sentence stem is "You will learn how to…" and it ties in specific procedural skills that students will acquire throughout the investigation, including the skills to "evaluate the sine, cosine, and tangent functions for any angle" and "solve equations involving trigonometric functions."[71] The

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[69] Cuoco, Al (2009). *Algebra 2: Center for Mathematics Education Project* (pp. 680-777). New Jersey: Pearson Education, Inc.
[70] Ibid. (p. 688).
[71] Ibid.

third sentence stem is "You will develop these habits and skills…" and it lists mathematical problem-solving and thinking skills that will be of significant value to students when they need to apply their learning to real-world situations both inside and outside the classroom. Students will be able to "extend the sine, cosine, and tangent functions carefully, in order to preserve key properties" and "use logical reasoning to find all possible solutions of a trigonometric equation."[72]

After using the three types of sentence stems to orient students on what they will be learning, each investigation has a section titled "For You to Explore" that is intended to activate students' prior knowledge and connect it to new ideas. The problems and questions that this exploratory section contains are heavily geared toward developing students' conceptual understanding because they ask students to explain what they are seeing and discovering in the guided exploration. We also mainly see conceptual understanding (with some procedural skill and fluency tied in) embedded in the two sections titled "Exercises: Practicing Habits of Mind" and "Developing Habits of Mind," which are a part of every lesson in the trigonometry investigations. The "Developing Habits of Mind" section develops students' critical thinking skills by guiding them in questioning mathematical definitions (e.g. $\tan\theta = \frac{\sin\theta}{\cos\theta}$ is only valid for $0 \le \theta < 90$)[73] and identifying key relationships between concepts (e.g. the Pythagorean Theorem and the trigonometric identity $\sin\theta^2 + \cos\theta^2 = 1$).[74]

While the "Exercises: Practicing Habits of Mind" and "Developing Habits of Mind" sections appear in every trigonometry lesson in Curriculum D, the other components of each lesson tend to vary a bit, which is different from the consistent structure in every lesson that we have seen in the previous three curricula. Each lesson is composed of a different variation of the following elements: definitions, examples, theorems, discussion questions, and practice problems. As with the other aspects of the text, conceptual understanding is strongly emphasized and paired with some procedural skill and fluency.

The assessments in Curriculum D include a mid-chapter test after the second investigation and a chapter test after the third investigation. As you saw depicted in the findings section of this paper, the assessment composition for Curriculum D is very similar to that of the direct instruction curricula because it incorporates a great deal of procedural skill and fluency in the quizzes and tests for the trigonometry unit. However, this focus on procedural skill and fluency that we see in Curriculum D's assessments does not necessarily align with the focus of its investigations in the trigonometry unit. In its investigations, we see that developing students' conceptual understanding is by far the main priority and goal of the text, so procedural skill and fluency plays a much smaller and more complementary role as it is interwoven with the conceptual understanding. The problems that are presented in the investigations ask students questions like: Can you think of a situation in which it would be better to use one method over another when solving for an angle measure, can you describe how the y-coordinate changes as the angle increases on the unit circle, and can you explain why the circle will pass through a specific point on the coordinate plane?[75] In Curriculum D's trigonometry assessments, over 50% of the questions are categorized as conceptual understanding + procedural skill and fluency, but their roles from the investigations are switched because conceptual understanding is now

[72] Ibid.
[73] Ibid. (p. 695).
[74] Ibid. (p. 685).
[75] Ibid. (pp. 684-689).

complementing procedural skill and fluency instead of vice versa as in the investigations. The problems that are presented in the assessments ask students to determine which quadrant an angle θ would be in given certain conditions, simplify trigonometric expressions, sketch angles in standard position, and use the sketch to find sin(θ), cos(θ), and tan(θ).[76]

Where do we see real-world applications in Curriculum D? We see brief references and connections to real-world applications throughout the text, but we do not see students actually having to engage with these connections between trigonometric concepts and the real-world. In the trigonometry unit for Curriculum D, neither the lesson exercises nor the chapter assessments have problems with real-world applications. However, real-world applications are referenced in the two paragraphs at the very beginning of the unit that introduce students to the study of trigonometry. The text explains that trigonometry is not only about triangles, but it is also about waves and oscillations. The text gives examples of where we see waves in the natural systems of our world, including the movement of light, sound, and molecules in a solid.[77] The curriculum then uses this information to make connections between trigonometry and different sciences, such as acoustics, optics, chemistry, and electrical engineering. By making these connections, the text is helping students to understand the real-world relevance of the seemingly abstract concepts that they will be learning about in the unit. Curriculum D begins the unit by providing a real-world frame of reference for the concepts that students will be learning about throughout the investigations; however, it is surprising that the curriculum does not include exercises that incorporate these real-world connections to further students' understanding of how to apply what they've learned in different settings.

What we do see throughout the unit on trigonometry are brief side-notes that reference real-world connections to the different concepts that are being presented. For example, when learning about what it means for the tangent function to be periodic, Curriculum D has a picture at the bottom of the page of a busy street with traffic lights and a description that reads, "The periods of the traffic lights are set to manage the flow of traffic."[78] The real-world reference to traffic lights helps students to conceptualize where they might see periodic functions come up outside of the classroom. Another real-world reference that the curriculum makes is to approximating the measure of physical landmarks that cannot be measured precisely, such as finding the width of a glacier. The text tells students that they can use triangle relationships to approximate the measurements of a glacier.[79] However, the text doesn't have students actually apply their knowledge of triangle relationships to measure the width of a glacier, which demonstrates a lack of procedural skill and fluency development in the text. By making brief connection to real-world applications of key trigonometric concepts throughout the chapter, Curriculum D is furthering students' conceptual understanding of how trigonometry can be used and applied in the world. While students will now have a better conceptual understanding of how trigonometry is used in the real world, they will not have any experience with using trigonometry to develop procedures to solve problems embedded in a real-life scenario.

Throughout the trigonometry investigations in Curriculum D, we have seen that developing students' conceptual understanding is the main focus of the text. <u>While there is a good amount of procedural skill and fluency interwoven with this conceptual understanding, there is a very apparent and serious lack of real-world applications in the curriculum, which leads</u>

[76] Ibid. (p. 775).
[77] Ibid. (p. 681).
[78] Ibid. (p. 724).
[79] Ibid. (p. 736).

us to conclude, as with the other three curricula, that Curriculum D's unit on trigonometry is not meeting the expectations of the Common Core State Standards for rigor since conceptual understanding, procedural skill and fluency, and applications are not being integrated with equal intensity in the mathematical content.

Conclusion: Where Do We Go From Here?

The central focus of my analysis was investigating how direct instruction and inquiry-based learning curricula integrate rigor into their divergent approaches to presenting content to high school mathematics students, based on the expectations of the Common Core State Standards. As we have discovered so far, direct instruction and inquiry-based learning curricula use methods that are seemingly similar as well as distinctly different to integrate rigor into their respective units on trigonometry. However, none of the four curricula that I analyzed for this project actually meet the expectations for rigor set by the Common Core State Standards because none of them pursue conceptual understanding, procedural skill and fluency, and applications with *equal* intensity.[80] The direct instruction curricula focus more on procedural skill and fluency while the inquiry-based learning curricula focus more on conceptual understanding. Curriculum C integrates real-world applications throughout its investigations and exercises, but the other three curricula only integrate a minimal amount of application problems.

Clearly, each curriculum has its own strengths, weaknesses, and challenges, but where does that leave us? More specifically, where do mathematics educators go from here? As you will see in the next section, mathematics educators can synthesize the different components of direct instruction and inquiry-based learning curricula into a sequence of lesson plans that utilizes the assets of both teaching styles in order to effectively present new content to students in an active and structured learning environment. While combining the strengths of two divergent approaches to teaching requires time, intentionality, and access to both types of curricula, it is a practical and impactful step that we as educators can take in order to more effectively differentiate our instruction with the goal of making the mathematical content more accessible to our students.

[80] *High School Publishers' Criteria for the Common Core State Standards for Mathematics* (p. 3).

Synthesized Lesson Plan

Lesson Day 1	Activity description/Teacher does	Students do
Title	Inquiry-Based Learning on the Connection between Similarity and Side Ratios in Right Triangles	
Standard	CCSS.MATH.CONTENT.HSG.SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.[81]	
Central Focus (CF)	Students will construct their knowledge of the connection between similarity and side ratios in right triangles as properties of the angles in the triangle in order to solve problems involving right triangles using trigonometric functions.	
Academic Language	Opposite, adjacent, and hypotenuse side lengths in right triangles, similarity, AA criterion for similarity, corresponding angles, acute angles, ratios, sine, cosine, and tangent. *Understand* and *make connections* between similarity and side ratios in right triangles.	
Learning Target (LT)	Students will understand that, by similarity, side ratios in right triangles are properties of the angles in the triangle.	
Instruction (e.g. inquiry, preview, review, etc.)	Teacher reviews prerequisite knowledge with students by giving them the following entry task: • The teacher draws three pairs of triangles on the whiteboard. One pair is congruent (and thus similar), one pair is similar, and one pair is not similar. • The teacher asks the students to individually determine whether each pair is or is not similar. • The teacher has students share their answer with a partner. ○ Teacher listens to the students as they pair-share during the entry task. Teacher listens for student understanding of what it means for two triangles to be similar. • The teacher randomly calls on students to share and justify their answer to the entry task.	Students complete the entry task individually. Students pair-share with a peer. Students participate in the whole class discussion of the entry task.

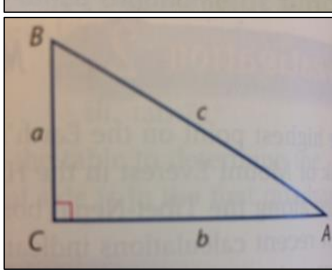
[81] National Governors Association Center for Best Practices, and Council of Chief State School Officers (2010). *Common Core State Standards for High School Mathematics* (p. 77). Washington D.C.: National Governors Association Center for Best Practices and Council of Chief State School Officers.

	o The teacher asks if the class agrees with the answers given. Why or why not? o What does it mean for two triangles to be similar? Teacher introduces the learning target for the lesson. Teacher introduces the Inquiry-Based Learning Lab and reviews the expectations for group work with the class. • Include everyone in the group work. • Ask questions. • Stay engaged in the group work. Teacher introduces the names for the sides of right triangles from a specific reference angle by drawing a right triangle on the whiteboard, identifying a reference angle, and labeling the sides as Opposite, Adjacent, and Hypotenuse.	Students write down the learning target. Students listen to the lab instructions and participate in reviewing the expectations for group work. Students listen as the teacher explains the Opposite, Adjacent, and Hypotenuse math convention.
Practice Activity or Support	Teacher assigns students to mixed ability groups of four and gives every student a ruler, a protractor, and the inquiry-based learning lab handout.[82] Teacher gives each group four similar right triangles, but every group's right triangles have two different angle measures (e.g. 30-60-90, 45-45-90, 10-80-90, 17-73-90, 26-64-90, 32-58-90). Teacher does not tell the students that their group's triangles are similar. Teacher walks around to each group and monitors their progress toward the learning target. Teacher will listen for: • Students using the academic language correctly. • Students making correct observations about the connection between similarity and side ratios in right triangles as properties of the angles in the triangle. • Students answering the questions on the lab correctly. • All students having a voice in their group discussion. Answers: 1) All of the triangles have the same angle measures.	Inquiry-Based Learning Lab Students will work in mixed ability groups of four to construct their knowledge of the connection between similarity and side ratios in right triangles as properties of the angles in the triangle. Every student in each group selects one of the four similar right triangles that were given to their group and uses their protractor to measure the two unknown angle measures. Students answer the following questions:

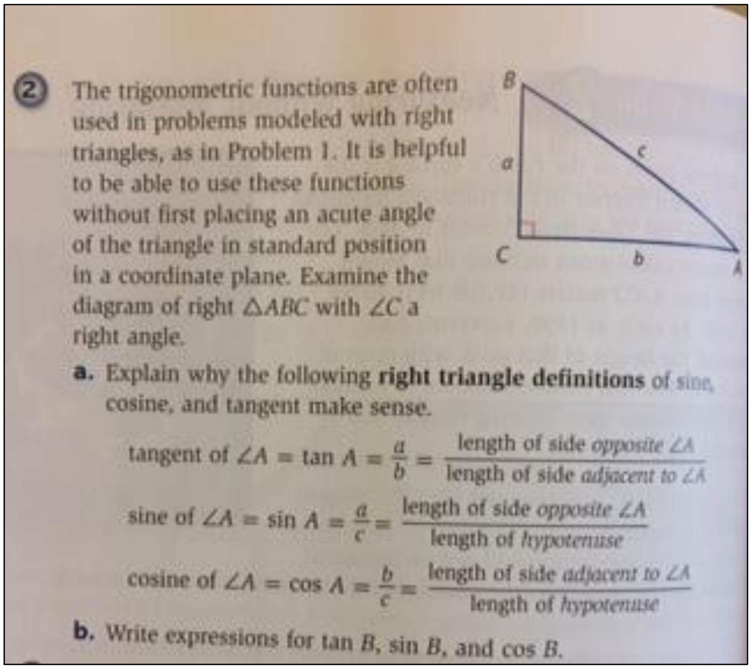
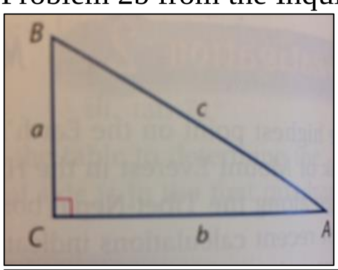
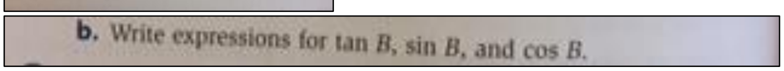
[82] HCPSS Secondary Mathematics Office (v2.1); adapted from: Leinwand, S. (2009). *Accessible mathematics: 10 instructional shifts that raise student achievement*. Portsmouth, NH: Heinemann.

	2) All of the triangles are similar. 3) 180°	1) What do you notice about the angle measures for all of the triangles in your group? 2) What conclusion can you draw from this and why? 3) What is the sum of the angle measures for each triangle in your group?
	Answer: 1) If we multiply each of the side lengths of one triangle by the same constant number, then we get the side lengths of a similar triangle.	Every student measures the 3 side lengths of their triangle, and then compares the side lengths of their triangle with the side lengths of their group members' triangles. 1) What observations can you make about the side lengths of similar triangles?
	Answers: 1) Opposite/Hypotenuse; Adjacent/Hypotenuse; Opposite/Adjacent; Hypotenuse/Opposite; Hypotenuse/Adjacent; Adjacent/Opposite. 2) The opposite side length would be the same as the hypotenuse.	Choose one corresponding angle in your group's triangles (not the right angle) as the reference angle and construct ratios for each triangle using the opposite, adjacent, and hypotenuse side lengths. 1) What ratios do you get for each triangle? 2) Why can't you choose the 90° angle to be the reference angle?
	Answers:	Use the other corresponding angle in your group's triangles as the reference angle and construct ratios for each triangle using the opposite, adjacent, and hypotenuse side lengths.

	1) Opposite side for the first reference angle equals the adjacent side for the second reference angle. Adjacent side for the first reference angle equals the opposite side for the second reference angle. Hypotenuse for the first reference angle equals the hypotenuse for the second reference angle. 2) By similarity, side ratios in right triangles are properties of the angles in the triangle.	1) How do these results compare to the ratios you got using the other angle as the reference angle? 2) What can you conclude about similarity and side ratios in right triangles?

Informal Assessment	Teacher evaluates the students' progress toward the learning target by observing their small group work on the Inquiry-Based Learning Lab.
	
	Teacher debriefs the Inquiry-Based Learning Lab with the whole class.
	• Teacher asks all of the groups to share the angle measures of their group's similar triangles and their findings from the lab with the class.
	    ○ What is similar about the findings for all of the groups?
	    ○ What can you conclude about similarity and side ratios in <u>all</u> right triangles?
	    ○ Teacher measures the students' progress toward the learning target by their responses.
	    ○ Teacher directly explains that by similarity, side ratios in right triangles are properties of the angles in the triangle.
	• Teacher presents the students with the following trigonometric ratios:[83]
	$\text{tangent of } \angle A = \tan A = \frac{a}{b} = \frac{\text{length of side } \textit{opposite } \angle A}{\text{length of side } \textit{adjacent to } \angle A}$
	$\text{sine of } \angle A = \sin A = \frac{a}{c} = \frac{\text{length of side } \textit{opposite } \angle A}{\text{length of } \textit{hypotenuse}}$
	$\text{cosine of } \angle A = \cos A = \frac{b}{c} = \frac{\text{length of side } \textit{adjacent to } \angle A}{\text{length of } \textit{hypotenuse}}$
	
		Students work in small groups to complete the Inquiry-Based Learning Lab.
		
		Students share their group's findings with the class and listen to other groups' findings.
		
		Students copy the trigonometric ratios down in their math notebooks.

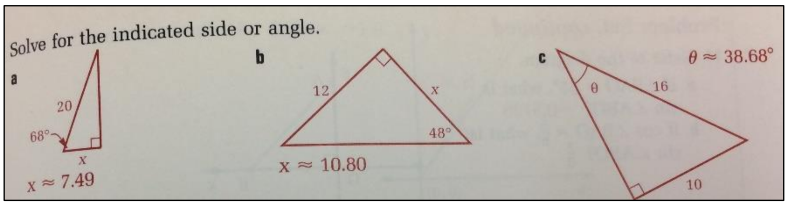
[83] *Core-Plus Mathematics (Course 2): Contemporary Mathematics in Context* (p. 468).

	• Teacher explains that they will learn more about these trigonometric ratios and why they are important in the next four lessons. Teacher tells the students that these ratios should be memorized.	
Closure Assessment of Student Voice	Exit Ticket: Problem 2a from the Inquiry-Based Learning Curriculum:[84]  (2) The trigonometric functions are often used in problems modeled with right triangles, as in Problem 1. It is helpful to be able to use these functions without first placing an acute angle of the triangle in standard position in a coordinate plane. Examine the diagram of right $\triangle ABC$ with $\angle C$ a right angle. **a.** Explain why the following **right triangle definitions** of sine, cosine, and tangent make sense. tangent of $\angle A = \tan A = \frac{a}{b} = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$ sine of $\angle A = \sin A = \frac{a}{c} = \frac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}}$ cosine of $\angle A = \cos A = \frac{b}{c} = \frac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}}$ **b.** Write expressions for tan B, sin B, and cos B.	Students complete the exit ticket.
Homework	Problem 2b from the Inquiry-Based Learning Curriculum.[85]   **b.** Write expressions for tan B, sin B, and cos B.	Students complete the homework and begin memorizing the trigonometric ratios.

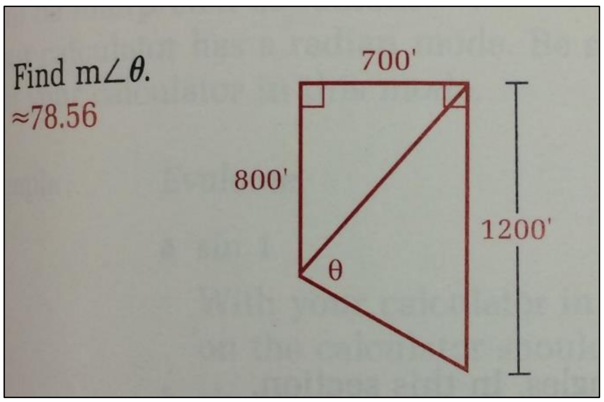
[84] Ibid.
[85] Ibid.

Lesson Day 2	Activity description/Teacher does	Students do
Title	Using Trigonometric Functions to Solve Problems involving Right Triangles	
Standard	CCSS.MATH.CONTENT.HSG.SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.[86]	
Central Focus (CF)	Students will construct their knowledge of the connection between similarity and side ratios in right triangles as properties of the angles in the triangle in order to solve problems involving right triangles using trigonometric functions.	
Academic Language	Trigonometric functions, inverse trigonometric functions, sine, cosine, tangent, opposite, adjacent, hypotenuse, right triangles. *Use* trigonometric functions to solve problems involving right triangles.	
Learning Target (LT)	Students will solve for angle measures and side lengths in right triangles using trigonometric functions.	
Instruction (e.g. inquiry, preview, review, etc.)	Entry task: the teacher draws a right triangle on the whiteboard and labels the three sides and angles. The teacher instructs students to write trigonometric expressions using the definitions of sine, cosine, and tangent that they learned yesterday. The teacher listens to students' conversations about the entry task. The teacher listens for: • Correct answers (the entry task is a review of the prior knowledge students need to have for this lesson). • Correct justification for their answers. The teacher randomly chooses a pair to share their answer and justification with the class. The teacher asks the class if they agree or disagree with that answer and justification. Did any group approach this entry	Students complete the entry task individually and write their answers in their math notebooks. Students discuss their answers to the entry task with a partner. Students justify their reasoning behind how they solved the entry task. Students participate in the class discussion. Students share ideas on how to

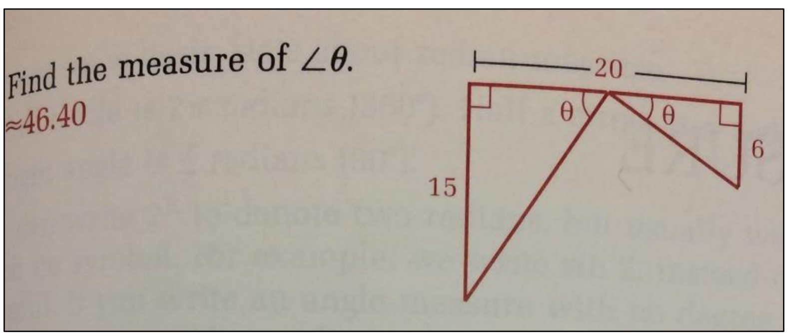
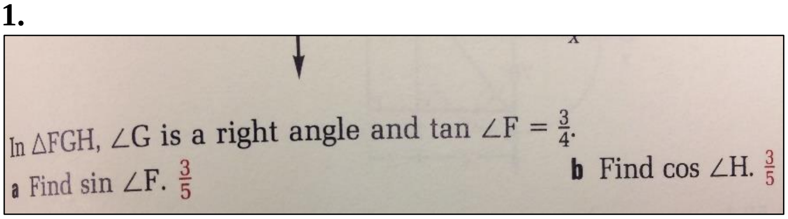
[86] *Common Core State Standards for High School Mathematics* (p. 77).

	task differently? Note that students could have chosen either angle to be their angle of reference, so there is more than one possible answer to this entry task. The teacher introduces the learning target. The teacher writes the following problem on the whiteboard:[87]  The teacher shows the students how to solve the problem using trigonometric functions. While the teacher solves this problem, she is intentional about vocalizing her thought process so that the students can follow her mathematical reasoning. The teacher introduces and emphasizes the academic language for this lesson as she uses it to explain her mathematical reasoning in solving these examples. When the teacher introduces the inverse trigonometric functions, she reviews what it means for two functions to be inverses with the class. The teacher also reminds the class to make sure that their calculators are in the correct setting when using the trigonometric functions (i.e. degrees vs. radians) and shows students how to use the trig and inverse trig functions on their calculators. The teacher tells students to check their work at the end of each problem by asking themselves whether or not their solution makes sense.	approach this entry task differently. Students write the learning target in their math notebooks. Students follow along with the lesson by taking neat and detailed notes in their math notebooks and by using their calculators when instructed. Students know that at the end of every unit, the teacher will grade their notebooks by looking for: • Learning targets copied down for each lesson. • Detailed notes on the concepts and examples that were discussed in class. • Neatness and organization. • Individuality (e.g. notes in the margin, color-coded notes, arrows connecting ideas, etc.)

[87] *Teacher's Edition Algebra 2 and Trigonometry* (p. 635).

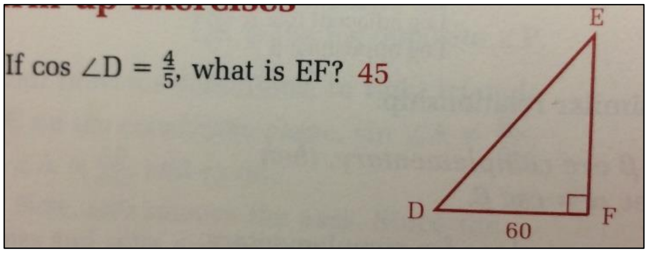
Informal Assessment	After the teacher completes each step in solving the examples above, she pauses to make sure that her students are following along. The teacher observes their nonverbal behavior to assess their understanding; the teacher clarifies the content when necessary. The teacher asks her students to look up at her when they are ready to move on to the next step in solving the problem.	Students stay engaged in the lesson by looking up at the teacher when they are ready to move on to the next step in solving the problem and by asking clarifying questions when needed.
Practice Activity or Support	The teacher divides the class into the mixed ability groups of four from day 1. The teacher writes the following problem on the whiteboard k times where k equals the number of groups.[88]  The teacher assigns each group a space on the whiteboard and instructs them to work together to solve the problem using trigonometric functions. The teacher encourages them to refer to their notes, to follow the examples that they just did as a class, and to use the academic language from the lesson in their small group discussions. The teacher assesses the students' progress toward the learning target by observing their small group work. The teacher listens for: • Correct use of the academic language for this lesson. • Students following the method that was outlined in the examples. • Every student actively participating and being included in the group work. • Students checking their solution to verify that it is correct.	Students work in small groups to solve the problem using trigonometric functions.

[88] Ibid. (p. 639).

	The teacher provides individual and small group instruction and support while students collaborate in small groups. The teacher randomly chooses a group to share their solution and justification for it with the class. The teacher asks the class if they agree or disagree with that answer and justification. Did anyone solve this problem differently? The teacher briefly summarizes what they learned in class today by connecting the learning activities to the learning target.	Students participate in the class discussion. Students listen to the teacher.
Closure Assessment of Student Voice	The teacher gives each student an exit ticket with the following question on it: Write about how you would solve this problem using trigonometric functions and justify your reasoning (don't actually solve it, just tell me the process that you would use to solve it and make sure that you justify why this process would work).[89]  	Students complete the exit ticket individually.
Homework	The teacher gives each student the following four problems for their homework assignment.[90] **1.**  In ΔFGH, $\angle G$ is a right angle and $\tan \angle F = \frac{3}{4}$. **a** Find $\sin \angle F$. $\frac{3}{5}$ **b** Find $\cos \angle H$. $\frac{3}{5}$	The students complete the homework and show all of their work.

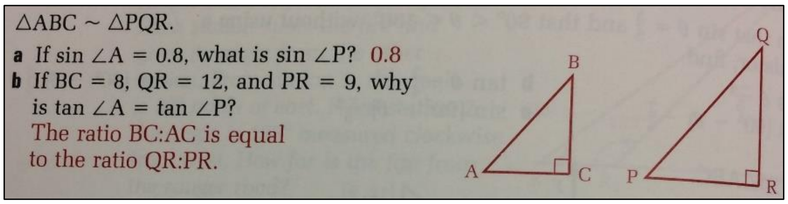
[89] Ibid.
[90] Ibid. (pp. 635-639).

2.



If cos $\angle D = \frac{4}{5}$, what is EF? 45

3.

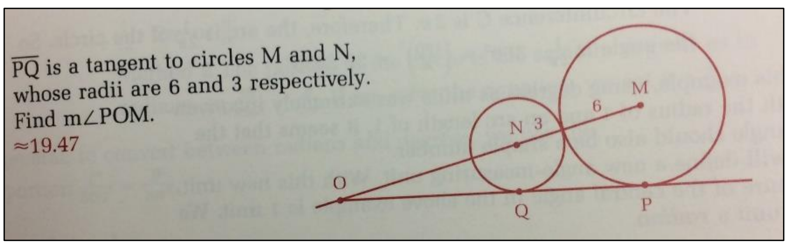


$\triangle ABC \sim \triangle PQR$.

a If sin $\angle A = 0.8$, what is sin $\angle P$? 0.8

b If $BC = 8$, $QR = 12$, and $PR = 9$, why is tan $\angle A$ = tan $\angle P$?
The ratio BC:AC is equal to the ratio QR:PR.

4.



$\overline{PQ}$ is a tangent to circles M and N, whose radii are 6 and 3 respectively. Find m$\angle$POM.
≈ 19.47

The teacher tells the students that the first three problems will be graded based on whether or not they were solved correctly and all of the student's work was shown, but the fourth problem will be graded on whether or not the student attempted it. The fourth problem is an extension of what was learned in class today.

Lesson Days 3-5	Teacher does	Activity Description/Students do
Title	Applications of Trigonometric Functions	
Standard	CCSS.MATH.CONTENT.HSG.SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.[91]	
Central Focus (CF)	Students will construct their knowledge of the connection between similarity and side ratios in right triangles as properties of the angles in the triangle in order to solve problems involving right triangles using trigonometric functions.	
Academic Language	Angle of elevation, angle of depression, angle of incidence, angle of reflection, acute angles, distance, altitude, angle measure, opposite, adjacent, hypotenuse, sine, cosine, tangent, trigonometric functions, inverse trigonometric functions. *Apply* knowledge to different real-world contexts and *analyze* the work of peers.	
Learning Target (LT)	Students will connect the concept of trigonometric ratios to real-world applications to further their understanding of how side ratios in right triangles are properties of the angles in the triangle.	
Instruction (e.g. inquiry, preview, review, etc.)	Teacher will write two review problems from the content that was learned the previous day on the whiteboard for students to complete as they enter the classroom. Teacher will instruct the students to share their answers to the entry task with a partner. ○ Teacher listens to the students as they pair-share during the entry task. Teacher looks for students applying the procedural skills that they learned in the previous lesson to the problems in the entry task.	Students will complete the entry task individually. Students pair-share.

[91] *Common Core State Standards for High School Mathematics* (p. 77).

	Teacher will review the answers to the entry task with the entire class by calling on students to share and defend their answers.	Students participate in going through the entry task as a class.
Practice Activity or Support	Teacher introduces the learning target for the lesson. Teacher tells the class that they will be working in their groups of four from day 1 to solve six application problems on trigonometric ratios. The students will have two days to work in their groups on all six problems, and then on the third day, each group will present their solution to one of the problems to the class (every group member must participate in the presentation). However, they need to prepare presentations for all six of the problems because they won't know which one their group will be presenting on until the third day. The presentation should include the following elements: • Labeled picture of problem. • Explanation of how the group approached solving the problem. • Explanation of how their application problem connects to the Learning Target. • Detailed and neat solution sheet that shows their work for	Students work in groups of four on all six of the following application problems.[92] They prepare solutions and presentations for all of the problems, but they will only present one of them to the class on the third day. The students won't know which problem their group is presenting on until it is time to present. A. More people these days are exercising regularly. Exercise scientists measure the amount of work done by people in various forms of exercise so they can learn more about its effect. One popular form of exercise is walking on a treadmill. What features of a treadmill do you think would increase or decrease the amount of work done by the walker? One index that exercise scientists use is the percent grade of the treadmill. Percent grade is computed as 100 multiplied by the sine of the measure of the angle of elevation θ of the treadmill. Suppose θ is in standard position. Compute the percent grade of a two-meter treadmill with a vertical rise of 0.25 meters and of 0.33 meters. How do you think the percent grade is related to the amount of work a person does on a treadmill? B. Steep hills on highways are the scourge of long-distance bikers. To measure the percent grade of a section of highway, surveyors use transits to estimate the average angle of elevation (or inclination) over a measured distance of highway. Then the percent grade is computed as 100 multiplied by the sine of the measure of the angle of elevation. If you ride down a straight 3-mile section of highway that has an 8% grade, how far do you drop vertically?

[92] *Core-Plus Mathematics (Course 2): Contemporary Mathematics in Context* (pp. 474-477).

	each part of the problem. • Correct mathematical reasoning and procedures. • Correct interpretation of the application problem. • Find one more question to ask about the problem and solve it as a group. Teacher gives students a copy of the rubric to reference as they work on their application problems (see page 36). Teacher explains what the rubric means by high level cognitive demand for the question that each group poses to themselves. Teacher reviews the expectations for group work with the class. • Include everyone in the group work. • Ask questions. • Stay engaged in the group work. Teacher evaluates the students' progress toward the learning target by observing their small group work on the application problems. Teacher listens to students' conversations and helps guide them toward solving their application problems.	If the angle of inclination of a 2-mile section of straight highway is about 4°, what is the percent grade? C. In Fort Recovery, Ohio, there is a monument to local soldiers who died in battle. Mr. Knapke, a teacher at the local high school challenged his class to find as many ways as they could to measure the height of the monument indirectly. Pedro, whose eye level P is 5.8 feet, proposed a novel solution. He placed a mirror M on the ground 45 feet from the center of the monument's base and then moved to a point 2.6 feet further from the monument where he could just see to the top of the monument in the mirror. He recalled from his earlier studies that the angle of incidence and the angle of reflection are congruent. Draw a diagram and show all of the given information. Figure out how Pedro found the height of the monument. What is the height? Describe another method to find the height of the monument. D. A survey team was asked to measure the distance across a river over which a bridge is to be built. They set up a survey post on their side of the river directly across from a large tree on the other side. Then they walked downstream a distance that they measured to be 400 meters. From the downstream position, they sighted the survey post and then rotated their calibrated transit to the tree to find the sighting angle to be 31°. Determine the distance directly across the river, that is, from the survey post to the tree on the opposite bank. Determine the distance from the surveyors' sighting point to the tree on the opposite bank. E. From the eye of an observer at the top of a cliff 125 meters from the surface of the water, the angles of depression to two sailboats, both due west of the observer, are 16° and 23°. Calculate the distance between the sailboats. F. Commercial aircraft usually fly at an altitude between 9 and 11 kilometers (about 29,000 and 36,000 feet). When an aircraft is landing, its gradual descent to an

		airport runway occurs over a long distance. Assume the path of descent is a line. Suppose a commercial airliner begins its descent from an altitude of 9.4 km with an angle of descent of 2.5°. At what distance from the runway should the descent begin? Suppose a commercial airliner flying at an altitude of 11 km begins its descent at a horizontal distance 270 km from the end of the runway. What is its angle of descent? The cockpit cutoff angle of an airliner is the angle formed by the pilot's horizontal line of sight and her line of sight to the nose of the plane. Suppose a pilot is flying an aircraft with a cockpit cutoff angle of 14° at an altitude of 1.5 km. In her line of sight along the nose of the plane, she sights the near edge of a lake. How far is she from the edge of the lake, measuring along her line of sight? What is the horizontal distance to the near edge of the lake?
Summative Assessment	Teacher chooses a different group to present on each of the application problems. Teacher assesses the small group presentations. Teacher looks for evidence of students' conceptual understanding, procedural skill and fluency, and understanding of their application problem in alignment with the standard for the 5-day lesson plan. Teacher assigns grades based off of the rubric on the next page and peer feedback.	Students present in small groups on the application problem that their teacher assigns to their group on presentation day. When students are not presenting, they are evaluating the presentations of their peers using the Applications in Trigonometry Rubric on the next page.

Applications in Trigonometry Rubric	**Excellent 4**	**Good 3**	**Needs Improvement 2**	**Poor 1**	**Your Score**
Labeled picture of the problem	Picture is neat, accurate, and labeled.	Picture is somewhat neat, accurately drawn, and labeled.	Picture is somewhat inaccurate.	Picture is inaccurate.	
Explanation of how the group approached solving the problem.	Explanation is detailed, thorough, and shows clear evidence of higher-level cognitive thinking.	Explanation is somewhat detailed and shows clear evidence of mathematical reasoning.	Explanation is unclear and needs more details.	Explanation is not given.	
Explanation of how their application problem connects to the Learning Target.	Explanation is detailed, thorough, and shows clear evidence of higher-level cognitive thinking.	Explanation is somewhat detailed and shows clear evidence of mathematical reasoning.	Explanation is unclear and needs more details.	Explanation is not given.	
Detailed and neat solution sheet that shows their work for each part of the problem.	Solution sheet is easy to read and follow and it shows all of the work and explains the mathematical reasoning behind each step in the solution.	Solution sheet is easy to read and follow, but it misses a couple of details that should be included in their work and reasoning.	Solution sheet is not easy to follow and it does not show all of the steps the group used to reach their solution.	Solution sheet is not easy to follow and it does not show adequate evidence of work or reasoning.	
Correct mathematical reasoning and procedures.	The method used to solve the problem works and is efficient. The answer is correct.	The method used to solve the problem works, but it is inefficient. The answer is correct.	The method used to solve the problem will work, but it was not performed correctly.	The method used to solve the problem will not work.	
Correct interpretation of the application problem.	Group demonstrates a strong understanding of their problem.	Group demonstrates an adequate understanding of their problem.	Group did not demonstrate a good understanding of their problem.	Group did not understand their problem.	
Find one more question to ask about the problem and solve it as a group.	Group asks a challenging question and demonstrates high level cognitive demand in their solution.	Group asks a challenging question, but does not demonstrate high level cognitive demand in their solution.	Group asks a straightforward question and solves it correctly.	Group asks a straightforward question, but does not solve it correctly.	
Total Points					**/ 28**

Reflection

My five-day lesson plan synthesizes different components from the direct instruction and inquiry-based learning curricula, and it integrates conceptual understanding, procedural skill and fluency, and applications into the differentiated learning activities. Day One is structured around an inquiry-based learning model that has students use similar right triangles to discover the relationship between the side ratios and angles of right triangles. For the majority of the lesson, students work in groups while the teacher circulates the classroom. Then, the teacher facilitates a whole-class discussion in which each group shares their findings with the rest of the class. The teacher asks the class to identify what is similar about each group's results in order to guide them toward concluding for themselves that side ratios in all right triangles are properties of the angles in the triangle by similarity. At the end of the lesson for Day One, the teacher switches to a model of direct instruction to introduce the students to trigonometric ratios. The purpose of this switch is to ensure that the students will walk away from the lesson having arrived at the intended concepts. Furthermore, the students are instructed to memorize these trigonometric ratios, which constitutes a lower-level cognitive demand.[93] However, this lower-level demand is transformed into a higher-level cognitive demand in the lesson plans that follow in this sequence.

Day Two is based off of a direct instruction approach to teaching students about how to solve for angle measures and side lengths in right triangles using trigonometric functions. In this lesson, the teacher demonstrates the procedure for using trigonometric functions to solve problems with right triangles while the students follow along by taking notes. Then, students are given the chance to practice following the procedure that their teacher outlined for them in both small group and individual settings. While this method of instruction is efficient in transmitting the knowledge of how to procedurally use the trigonometric functions to solve problems with right triangles, the question of whether or not the students are able to conceptually understand the procedures that they are following depends on the knowledge that they constructed for themselves in Day One about the relationship between the side ratios and angles of right triangles. Essentially, in the lesson plan for Day Two, we see a greater emphasis on what the teacher knows and is trying to communicate to the students than on what the students know and still need to learn, which, according to Healey, aligns with the attributes of a direct instruction curriculum.[94]

The lesson plan for Days Three, Four, and Five synthesizes the procedural knowledge that students learned in Day Two and the conceptual knowledge that students constructed in Day One by having students make connections between trigonometric functions and real-world applications through inquiry-based group work on an assigned set of problems. In this lesson, students are asked to apply what they learned through direct instruction in Day Two to figure out solutions to real-world problems without explicit guidance from their teacher. Since the students are not being directly taught how to solve these application problems, this method of instruction is perhaps less efficient in terms of how much class time is spent on six problems; however, it is effective in developing students' mathematical problem-solving abilities as well as their ability to use trigonometric functions in different real-world contexts. Overall, direct instruction and

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[93] Smith and Stein, "Selecting and Creating Mathematics Tasks: From Research to Practice" (p. 348).

[94] Healey, "Linking Research and Teaching: Exploring Disciplinary Spaces and the Role of Inquiry-Based Learning" (p. 70).

inquiry-based learning are both key components of my five-day lesson plan sequence; and while they both bring a different flavor to lesson plans in general, direct instruction and inquiry-based learning can be synthesized in a manner that effectively and efficiently differentiates mathematics instruction.

The first lesson in my five-day sequence is primarily focused on developing students' conceptual understanding of the relationship between trigonometric functions and right triangles. In the second lesson, procedural skill and fluency is tightly interwoven with the conceptual understanding from Day One as students learn how to procedurally solve for angle measures and side lengths in right triangles using trigonometric functions. The final three lessons are centered on applications of trigonometric functions. The application problems that students work on solving in those lessons intentionally build off of the conceptual understanding and procedural skill and fluency that were developed in the first two lessons. Thus, all three components of rigor are tightly interwoven in the final three days. Conceptual understanding is seen in isolation during the first lesson and in conjunction with procedural skill and fluency during the second lesson. When real-world applications of trigonometric functions are introduced to students in the third lesson, we see an integration of conceptual understanding, procedural skill and fluency, and applications in each problem of the assigned set.

Similar to what was observed in my analysis of the direct instruction and inquiry-based learning curricula, the three components of rigor may not always be seen with equal intensity in every individual lesson plan. However, when we examine a sequence of lesson plans, it has been deemed essential by the Common Core State Standards that conceptual understanding, procedural skill and fluency, and applications are all integrated with equal intensity into the various modes of instruction whether it be direct or inquiry-based.[95] My analysis and synthesis of direct instruction and inquiry-based learning curricula is relevant to the ongoing discussion on how mathematics should be taught because it illustrates a way in which strategies from both types of curricula can be realistically and rigorously implemented into a sequence of lessons. There are strong arguments for both sides of the direct instruction versus inquiry-based learning debate, but my project shows mathematics educators that we do not necessarily have to choose one system of teaching over the other; rather, as educators, we can effectively differentiate our instruction by synthesizing the assets that are present in both of these curriculum styles.

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[95] *High School Publishers' Criteria for the Common Core State Standards for Mathematics* (p. 3).

Glossary

Applications—a component of rigor that is focused on connecting mathematical ideas to contextual problems by teaching students how to use content knowledge and skills to solve real-world problems.[96]

Common Core State Standards—an educational initiative that specifies the expectations for what students in grades K-12 should learn at each grade level.[97]

Conceptual Understanding—a component of rigor that is focused on developing students' understanding of key mathematical concepts.[98]

Direct Instruction—a traditional style of teaching in which knowledge is simply and directly communicated by the teacher to the students.[99]

Inquiry-Based Learning—a non-traditional style of teaching in which students actively construct their own knowledge through investigation.[100]

Procedural Skill and Fluency—a component of rigor that is focused on providing students with opportunities to practice managing computational details with algebraic operations in a way that furthers students' conceptual understanding of important mathematical principles.[101]

Rigor—to pursue, with equal intensity, the three aspects of rigor: conceptual understanding, procedural skill and fluency, and applications.[102]

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[96] Ibid. (p. 10).
[97] Ibid. (p. 1).
[98] Ibid. (p. 9).
[99] Kirschner, Sweller and Clark (2006). "Why Minimal Guidance During Instruction Does Not Work: An Analysis of the Failure of Constructivist, Discovery, Problem-Based, Experiential, and Inquiry-Based Teaching" (p. 75).
[100] Ibid.
[101] Ibid.
[102] Ibid. (p. 4).

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Appendix: Faith and Learning

Ambiguity & Persistent Faith: An Essay on What Kind of Scholar Am I?

Education and hard work have always been the "religion" that my family follows. The central belief of it is that if you work hard and do well in school, then you will have a bright future. I was taught that education would be the key to my future, and that this key would unlock all sorts of possibilities for me. Another characteristic besides hard work that has always been highly valued by my family is independence, namely, the ability to support oneself. I became a Christian (non-denominational) when I was in eighth grade, and while my faith was very important to me, I didn't understand how it could be connected to my work as a student, which resulted in a complete disassociation between the spiritual and academic realms of my life. Looking back on it now, I would describe my Christian faith during high school as inherently individualistic, which is a description that still, to a certain extent, applies to my faith today. Of course, seeing myself and my faith from an individualistic perspective is very reflective of the Western culture in which I have always lived, but it is also reflective of my desire to be a "self-sufficient" Christian. I hope that desire sounds a little off target to you too because I have since learned that by identifying as a Christian, I am actually acknowledging that I am insufficient on my own and that I need the grace and guidance of God in every area of my life. While this acknowledgement may sound quite lofty and cliché, it encompasses the core of a truth that I find so refreshing to internalize in the midst of a society that is obsessed with productivity: my worth is not measured by my accomplishments.

One particularly transformative experience that helped me to see my need for God and community in my life was when I studied at Stanford University during the summer before my last year of high school. I had never felt so alone or so afraid of failing before, yet one of the greatest lessons that I learned from my experience at Stanford was how to see myself apart from my academic successes and failures. I have always felt as if my grades define a significant part of who I am because I have always understood them to be reflective of my work ethic—and I pride myself in being a very hard worker. While caring about grades and being a hard worker both sound like great attributes, I have oftentimes taken them to such unhelpful extremes that I do not know how to define myself without them. However, I am learning how to see past such a narrow-minded view of myself so that I am no longer defined by my academic accomplishments, but by something much more intangible and integral to who I am as a person of faith and hope in a world of ambiguity and uncertainty.

After much reflection, I have identified three faith commitments that are at the very core of who I am and who I want to be as a scholar. The first is my commitment to being "all in" when it comes to Christian scholarship. I have struggled for a long time with how to breach the gap between my academic work and my spiritual beliefs, and while I am still learning the ways in which scholarship and Christianity can intersect in my life, I am committed to eliminating the separation between them. In *Scholarship & Christian Faith: Enlarging the Conversation*, Douglas and Rhonda Jacobsen deconstruct the expectation that many Christian scholars may have about finding "neat and tidy" answers to their questions about how faith and scholarship can and should interact.[103] They argue that when it comes to studying pertinent questions of

[103] Jacobsen, Douglas and Rhonda Jacobsen (2004). *Scholarship & Christian Faith: Enlarging the Conversation*. New York: Oxford University Press (p. 21).

Christian scholarship, all understanding is "tentative and fragile," both of which are seemingly contradictory descriptors of how most scholars may perceive scholarship.[104] Essentially, the Jacobsens believe that connecting faith to scholarship and scholarship to faith is an ongoing process. Furthermore, they view Christian scholarship as a way through which all aspects of one's life can be connected, including faith and learning. I recognize that while I may prefer "neat and tidy" answers to my questions about the intersection between faith and learning, I must be willing to embrace the ambiguity and fragility that characterize Christian scholarship because failure to do so could ultimately lead me to a state of self-deception and an unwillingness to challenge my own convictions.

My second faith commitment is to being transparent with God. At first, this commitment may sound a little silly because God already knows everything about me, but I can tell you that it really matters and makes a difference in how I conceptualize what it means to be a Christian scholar. I think the most important consequence of having transparency before God is that it makes my relationship with God unscripted. No longer am I pretending that being both a Christian and a scholar is easy and that there are not any challenging questions and situations that arise from being a Christian scholar in a predominantly secular world. Instead, I am committed to taking my Christian faith out into the world in a way that is unscripted and unchartered—but for the record, let it be known that I would prefer an explicit script and path to follow.

My third faith commitment is to accepting the mystery and ambiguity that characterizes the life of a Christian scholar. In Chapter Two of *Scholarship & Christian Faith: Enlarging the Conversation*, Douglas and Rhonda Jacobsen highlight the work and theories of Nancey Murphy, a Christian scholar who rejects the modern idea that belief equals knowledge if and only if it is based on objective facts and valid logic. Murphy argues that knowledge is unavoidably complex and that everything is dubitable, which doesn't make it sound all that appealing to me. She believes that true Christian scholarship has more to do with questions and less to do with answers, a characteristic that may make most scholars, including myself, feel rather uncomfortable. Murphy argues that it is essential for faith and scholarship to be connected, and she believes that Christian scholars can make this connection by allowing their learning and faith to interact; however, she acknowledges that when learning and faith interact, they may point in different directions. As a Christian scholar, I recognize the ambiguous nature of true knowledge; and so instead of ignoring the points of tension between Christianity and scholarship, I will strive to settle into the mystery without looking for an easy way out.

My faith background and commitments are the foundation for my work as a high school mathematics teacher. I want my students to receive an equitable and rigorous education and to feel academically empowered in my class. For my honors project, both of these hopes were important motivators in my decision to analyze two contrasting teaching strategies in mathematics curricula through a framework that evaluates how they meet the expectations for rigor in math education. Through my analysis of these different instructional strategies, I am hoping to gain insight that will help me in developing my pedagogical philosophy regarding how I will cultivate an equitable and rigorous learning environment in my classroom.

As a teacher, I want to encourage and require my students to take ownership for their own learning. One way in which I will do this is by setting high academic and behavioral expectations coupled with a high level of differentiated support for my students. I also want to acknowledge the fact that mathematics is a very challenging subject for many students, and I want to teach my future students to embrace the challenge and engage in the struggle of learning

[104] Ibid. (p. 22).

mathematics.

As a Christian scholar and prospective teacher, one important question for me to consider is how do I uphold my faith values in a secular education system? One of the greatest truths about teaching is that the potential impact teachers can have on the lives of their students cannot be overstated. Students remember their high school teachers. However, they remember more about who their teachers were to them and the examples that their teachers set for them than the subject matter that their teachers taught. This statement is not meant to minimize the importance of a rigorous curriculum; rather, it is meant to emphasize the importance of character when it comes to teaching. I think that the most impactful way to uphold my faith values in the classroom is by being a teacher who cares for her students in a way that exemplifies grace, dignity, and respect.

In *The Outrageous Idea of Christian Scholarship*, Marsden argues that it is imperative for Christian scholars to be a part of the "mainstream academy" and to not remove themselves from the rest of the academic world.[105] Christian scholars contribute an important perspective and attitude to the academic world, and so instead of feeling like we have to choose between assimilation and separation, Marsden believes that we should courageously integrate our theological beliefs into our various disciplines in ways that appropriately complement our academic endeavors. As a teacher, I will bring my Christian faith into the classroom every day through the ways in which I teach, care for, and interact with my students.

Similar to Paul Farmer in *Mountains Beyond Mountains*, I feel a strong calling to help others. While I may not be saving lives in the same sense that Farmer does as a doctor, I believe that I will have the opportunity to make a meaningful difference in the lives of my students by providing them with opportunities to learn and see that they are all capable of being scholars, regardless of what others have said and what they have previously believed about their academic abilities. Students are oftentimes labeled as academically gifted or challenged from a young age, and studies have shown that these labels can have a serious impact on how students view themselves as well as how they apply themselves academically. As a teacher, I want to disrupt this pattern of assigning students rigid labels that inappropriately define and limit their academic capabilities by instilling within my students a strong sense of self-worth and self-efficacy.

One of the most frustrating aspects of both my academic discipline and my commitment to the Christian faith is that there are no perfect formulas for how to be an effective teacher or a faithful Christian. I find this to be very frustrating because I really like to have formulas to follow—which may be one of the reasons why I am pursuing a degree in mathematics. While there is no clear-cut formula for being a Christian, the caveat to this statement is that Christianity in our Western culture does not always portray such an ambiguous message. I believe it is a common misconception by both Christians and non-Christians alike that there is a standard formula of sorts that makes a person a "good Christian." Some parts of this formula may include being baptized, going to church every Sunday, and reading the Bible every day.

Yet, in my opinion as well as Paul Farmer's, that is not enough. Farmer articulates this belief when he says to Tracy Kidder, "That's when I feel most alive…when I'm helping people."[106] As a Christian scholar, I believe that I am called to help people in whatever way God has equipped me to do so. While I may not know all that such a calling entails, I rest in the faith

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[105] Marsden, George M. (1997). *The Outrageous Idea of Christian Scholarship*. New York: Oxford University Press (p. 100).

[106] Kidder, Tracy (2009). *Mountains Beyond Mountains: The Quest of Dr. Paul Framer, A Man Who Would Cure The World*. New York: Random House (p. 38).

that God will be with me throughout the entirety of my journey as I seek to live out this calling to help others in some way. I believe that Christianity and scholarship must go together in my life, but I am still learning how exactly these two distinct facets of knowledge intersect in my academic discipline. All in all, Christian scholarship is challenging yet worthwhile, mysterious yet illuminating, and ambiguous yet fruitful.

I am a Christian scholar who is learning to live with ambiguity in a world that is even more intricate than I imagined while also seeking to uphold my faith values through a life committed to stewarding the intellectual gifts of others. My hope for my continued studies as a Christian scholar is that when uncertainty clouds the horizon, I will seek the truth in a way that is both accepting of ambiguity and persistent in my faith.