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Understanding the Beauty of Mathematics by Composing Claude Debussy's Syrinx into Mathematical Equations

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by

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Abstract

Mathematics is seen as cold and calculating, while music is seen as expressive and beautiful. By composing Claude Debussy's *Syrinx* in terms of mathematical equations, we will be using Fast Fourier Transforms in MATlab to turn our frequencies from time-frequency to frequency amplitude domain to find Fourier Coefficients. Afterwards we will take these coeffificents and use them to recreate the sound using sine equations. This process will allow us to dive deeper into the beauty of mathematics.

Music Intro

Note, that this project will include interdisciplinary aspects regarding music and mathematics. We will be using mathematical terms frequency, harmonics, and amplitude. Refer to .1 for more information.

Introduction Math

In the scope of this project, we will focus our attention on how to model music specifically for flute using mathematics. Before we continue, let's think of sinusoidal and co-sinusoidal waves in a different perspective. If we are in the Complex Plane $Re^{i\theta}$ where R is the radius and θ is the angle in radians in a trigonometric setting. Now note , we can think of points as a having an $X = Rcos(\theta)$ and $Y = Rsin(\theta)$
Observe that,

$$e^{i\theta} = cos(\theta) + isin(\theta)$$
$$Re^{i\theta} = Rcos(\theta) + iRsin(\theta)$$

Additionally, this can θ can vary with time which we will call $\theta = \omega t$ which than our equation becomes:

$$Re^{i\omega t} = Rcos(\omega t) + iRsin(\omega t)$$

Note that we will be thinking of $Re^{i\omega t}$ as a vector with magnitude R and spinning counterclockwise at *omega* radians per second. With this definition in mind we want to visualize sine or cosine as counter-rotating vectors. [?]. Furthermore, the only distinction between sine and cosine waves is the our definition of time origin. Consequently, we will be using mathematical function *sin* to represent our waves.

Intro to Fourier Analysis

Fourier Analysis is the study of how general functions can be decomposed into trigonometric functions with definite frequencies.

From this, we can take two different approaches to Fourier Analysis. Our first approach is using an Fourier Series which is normally taken with periodic functions continuous or discrete, and our second approach are Fourier Transforms which is applies to more general functions that are not necessarily periodic but can be either continuous or discrete. [?]

Fourier Series and Music

With this, we are now equipped with theoretical knowledge of how we can model sinusoidal waves. Recall, that sound waves are sinusoidal. Now we are able to decompose sinusoidal waves into different waves but are unable to extract the specific frequencies. With a Fourier Transforms we are able to take an audio signal such as a wav file, a common music file, and represent it in terms of frequency and amplitude which is called frequency-domain rather than the time-domain which is represented in time and volume.

From a mathematical perspective, notice we can have two different types of series: a finite series or infinite series. However, in the music world, we are unable to have infinite sound because it dampens and decays. Consequently, this leads to a finite Fourier Series which will lead us using the Discrete Fourier Transform (DFT).

When we find the Fourier Coefficients of our Discrete Transform we will be using the Fast Fourier Transform or FFT. Note that we choose to use the FFT for its efficiency. Similar to the DFT, it is a collection of trigonometric sums to some length N. The difference is that FFT factors in the weights of each of these terms with the use of periods and symmetries to effectively shorten how many sums actually affect the summations.

Now, we have figured out we will calculate these Fourier coefficients but we run into a problem of turning an infinite series to a finite series. Because of the periodic and symmetric form of sinusoidal waves, if we create a finite sums, we might run into the issue of not creating a full period. In this case, what our algorithm would do is to fill in our missing period by starting a new period over it such that we have extra information. In this case what we would have to do is use an spectral analysis technique called aliasing. This is where we filling our missing data by creating using other sinusoidal waves that have the same nodes as our original wave would have if it completed its full periodic cycle.

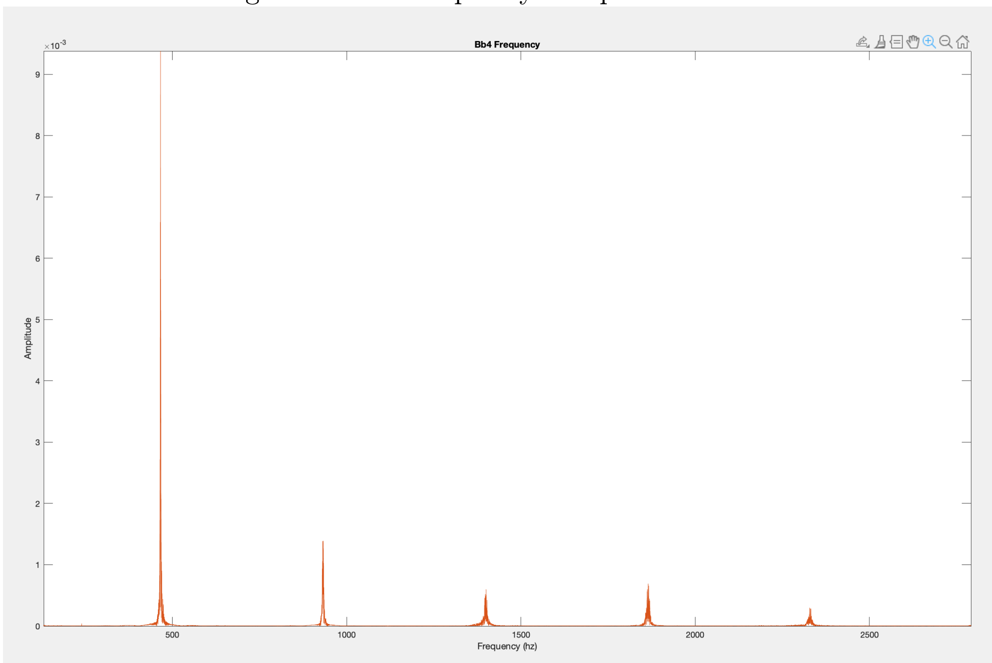
Mathematical Modeling Methods

Now, we are going to go emphasize certain key aspects of the project in relation to our methods.

To begin modelling Claude Debussy's *Syrinx*, we began by playing the notes on the flute that we wanted to model with. We played 15 different notes, and recorded them using Pro Tools. The notes we recorded were the notes that appeared in the first two measures of Syrinx: B flat 5, A5, B5, A flat 5, G5, A5, G flat 5, F5, E5, D flat 5, C6, B flat 4, F sharp 4, G4, and B4. After recording these notes, we downloaded their wav files .1 and exported it to MatLab. From here, we created scripts for each of the individual notes, and find Fourier Coefficients for each of the notes.

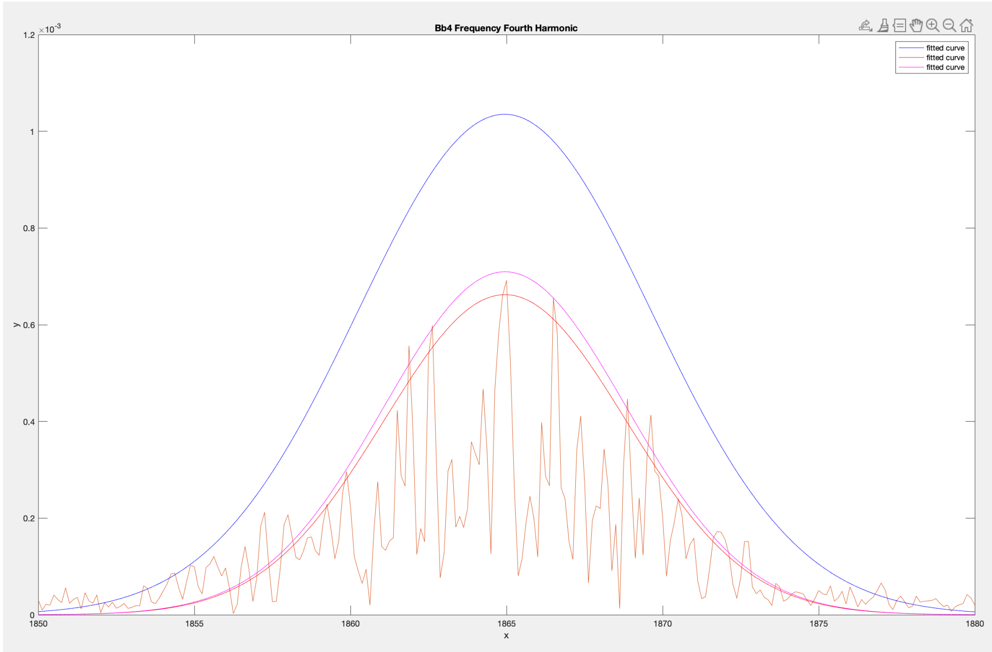
Our process will be graphing each note in frequency-amplitude and then model each of its harmonics. There are generally nine playable overtones on the flute, but there can be as many up to twenty-seven. [**?**] In the scope of this project, we will use four harmonics: the fundamental tone, an octave higher, a perfect fifth above and then two octaves above the fundamental. .1. We have made this decision to encompass some of the timbres of the flute, but also knowing that we would not been able to capture all of them due to time constraints. [**?**]

Figure 1: Bb4 Frequency- Amplitude Domain



After loading each individual wav file for each of the notes, we want to find the Fourier Coefficients, Therefore, we first need to take our notes from the time-frequency domain into the frequency-amplitude domain. 1 We do this by using our Fast Fourier Transform algorithm. Note that each peak in 1 represents an harmonic with the highest peak representing the fundamental tone. Remember we are only doing the first four due to that being the least number of harmonics to fully capture the flute timbre.

Figure 2: Bb4 Fourth Harmonic Modelling



After this, we then fit our four harmonics using Gaussian distributions and changed the interval we were looking at based off the harmonic we were observing. 2 It was a trial and error process that we repeated and there was an average 3 models per each harmonic which made 12 models for each note. In total, we have found 60 unique Fourier Coefficients for our 15 notes.

After finding our Fourier Coefficients, we needed to create time arrays which allowed us to graph in the time-frequency domain for each of our notes. These time arrays allowed us to map certain frequencies on a scale of time. Then from here, we were able to change our sampling rate to get more accurate readings. Lastly, for each individual note, we then compiled and created a sine equation which included the four harmonics with their respective frequencies and amplitudes.

After creating the sound, we changed the sampling rate to allow for variation which causes this idea of rhythm within the lines. This now results in our rendition of *Syrinx*, composed of code, mathematical equations and an audio file.

To see the full mathematical composition refer to .3

Now we will discuss the implications of this research by asking what is music seen as beuatiful while mathematics is not.

Defining Beauty

As expected, it will be hard to define beauty especially "mathematically beauty, but just as true of beauty of any kind - we may not know quite what we mean by a beautiful poem but does not prevent us from recognizing one when we read it?" [**?**]

Beauty is "the quality or aggregate of qualities in a person or thing that gives pleasure to the senses or pleasurably exalts the mind or spirit." [**?**]

From this definition, we can see that "exalting the mind and spirit" appeals to the beauty of math. For mathematicians, "exalting the mind" implies working through problems and figuring out greater truths. However, solving problems includes two aspects: the solution itself and the key ideas they communicate. A famous example, Euclid's Proof of the Infinitude of Primes, which states that given any collection of prime numbers there is at least one prime number. [?] The beauty of Euclid's proof is in it's simplicity and elegance of how he wrote the proof. So his solution is considered beautiful for just the presentation of the solution. Furthermore, the idea communicated in it in which there are an infinite number of primes is beautiful as well, because of it's implications in the creation of mathematical knowledge.

Aesthetic Subjectivism and Aesthetic Objectivism

Some will argue, that "Beauty is in the eye of the beholder, " while this may hold some substance, there exists some universal beauty standard we hold ourselves too. For instance, a musical performance from a fifth grade talent show versus the musical performance from professional musicians. Generally, without any objectivity this would equate both the the fifth grade musical performance with the professional musicians. There would be no way to objective standards to judge beauty. [?] We would describe this as Aesthetic Subjectivism. [?] To give a more formal definition, Aesthetic Subjectivism is the belief that aesthetic judgements do not state facts about the world, but merely reflect and observer's response to some aspect of the world. [?] Solely subscribing to Aesthetic Subjectivism can be problematic because it violates our common sense of value and superiority. For instance a Monet's *Water Lilies* are superior works of art compared to a child's water lily finger painting despite the parents' emotional attachments. Therefore there must be some claim to say that beauty can at least be partially judged objectively. We call this Aesthetic Objectivisim. For a more formal definition, Aesthetic Objectivism is the belief that aesthetic qualities are properties of the objects that are independent of an observer's awareness.

There is the argument that what if people can not find the beauty of mathematics or any subject. In this, think of Imannuel Kant's Critique of Judgement. "the judgement of taste is subjective." Meaning that mathematical beauty still exists regardless whether people can find the beauty in it or not. Granted, we should help others find this beauty, but we should not let others belittle it because they do not understand it.

Different Types of Mathematical Beauty

Music has been historically seen as beautiful; however mathematics is not. Mathematics is seen as cold and calculating. This is not true, people have "not had a chance to experience it, or they experience it but not recognized as the experience as mathematical. [?] Mathematics has been taught in a manner in manner which causes it to become formulaic. However, we have just rewritten *Syrinx* as a mathematical composition. Each note is composed of audio data that we analyzed into their respective frequencies and harmonics. We then rewrote the sound in terms of sinusoidal waves. Why do we consider beautiful but not music, especially since we have shown that music is just mathematics.

From here we will draw the comparison between the parallels the beauty of music to the beauty of mathematics. Additionally, we will further this exploration with the connection of music file.

Sensory Beauty

Now music is seen as beautiful because it is able to be experienced through the senses: the pleasantness of the sound and the emotions entailing it. Additionally music allows us to be in awe of what the melodies and harmonies that transpired. Musicians perform difficult technique passages with ease and are able to convey stories by creating sounds out of thin air. From here, musicians are able to take certain stories and ideas and bring a more universal truth. For example, in Tchaikovsky's *Romeo and Juliet Overture* he tells the story of Romeo and Juliet through music which can be alluded to more thematic universal truths. One of them being, the idea of labels and names which lead to our prejudices.

The idea of being experience music through the senses is called, sensory beauty. Sensory beauty not only applies to sound, but all the senses as well.

In math, this beauty is applied to patterned objects that we can experience with the sense: sight, touch and sound. [**?**]. Additionally, mathematical sensory beauty presents itself in nature and art.

In nature, mathematics is present through patterns. The famous example is the Fibonacci sequence. This sequence originates from how rabbits reproduced given that each pair has one male and one female. This one question resulted in seeing this pattern everywhere in nature. It is seen on the seeds of sunflowers, cauliflower, and pine-cones. Futhermore, it is seen storms and how they spiral and move. Even the human body follows this pattern. With our 1 nose, 2 eyes, 3 segments to each limb and 5 fingers. Mathematics is seeing and applying patterns in different contexts.

In art, mathematics is presented in the creation of parallel lines in artwork. In the Last Supper, the parallel lines on the ceiling are constructed by created an intersection point at infinity. This gives us the illusion of parallel when they don't look like it.

Moreover, in mathematics a famous example of sensory beauty is fractals. In essence fractals are repeated patterns that keep repeating despite zoomed in a person gets to it. The most famous of these examples include the Fibonacci Sequence, which is most commonly seen in nature. It is seen through Romanesco Cauliflower, Sunflower seeds, sea shells, and much more. Additionally, there is also the Koch's snowflake and snowflake sweep. Both of these are examples of fractals and once combined produce beautiful images that capture one's attention. [**?**]

Specifically in this project, we appeal to sensory beauty of math through sight and sound. We can see the physical representation of our equations, and we can also hear them.

There are many possible feelings and emotions can be tied to this: awe, confusion, intrigue, but we brought it to life by coding it. Granted the sound itself seems devoid of emotion, as it is a generated through the computer, but the creation of this project allows us to reconsider what it means to be beautiful. This speaks to the creation of knowledge and the technical ability to create a reproducible audio clip of a famous flute solo.

Wondrous Beauty

Additionally, music has this feeling of being in awe, this is called wondrous beauty. In mathematics and especially during this project, wondrous beauty can also be ascribed as sparking one's curiosity. In the creation of the audio file, there were many questions: is there a way to solve this problem to get my desired result. This was a constant throughout the entire process of creating a computer generated sound modelling music. This is one aspect in which math is beautiful, because it allows us to understand the how and why behind ideas.

While sensory beauty and wondrous beauty are related, each type of beauty possess the ability to be independent of one another. [**?**] In mathematics, the beauty lies in the idea and main concepts. Therefore certain equations can be be admired for their physical beauty but also for the content they contain. The beauty from my audio file, is pleasing to the senses but is also beautiful in it's ability to represent that music is mathematics written in a different way. Each individual note can be written as a sum of sinusoidal waves with each wave representing a different harmonic.

This aspect of mathematical beauty differs slightly from music, because music evokes awe and a sense of calm, while mathematics evoke curiosity and start the process of understanding the mechanics of the world. The reason for this difference is that music itself is about a temporary interpretation of the piece. This piece gets interpreted in different ways over the years, but the beauty is from a fleeting moment. In mathematics, the beauty is in the permanent truth of ideas and proof. The idea is permanent in which other mathematicians can build and expand upon this beauty in concrete permanent ways.

Insightful Beauty

To be warned, insightful beauty is where the stereotype of mathematics being cold and devoid of emotion originates because it focuses on the art of understanding through reason. This is called insightful beauty. [?] From here, this is where people understand and find solutions to their problems.

Musicians use reason and logic to figure out the counts of beats, to diagnose potential problems in the music, but this aspect of music does not get represented or even listed as one of the motivating factors to perform music.

However in math, this gets constantly gets applied. Figuring out the least amount of food to order to feed enough people, from figuring out how to get people into space. Note that, these insights can be instantaneous but also an appreciation over time. Because learning is a lifelong activity. Furthermore insightful beauty in mathematics is best exemplified through the art of proof in which we look for the simplest or most insightful proofs to truly understanding the mechanics of the world.

This is the power to communicate ideas effectively.

During this project, there were obstacles to overcome: figuring out the code to learn how to find the coefficients, figuring out the best model to accurately represent the amplitude and frequencies of the different harmonics, figuring out how to best reproduce the sound in a way that is similar to the original piece. Understanding higher level math such as Fourier Analysis to comprehend the theory that the computer is doing. We struggled throughout these troubles of problem solving, but we continued to preserve because there must be a solution and refused to give up. Because of this, "The beauty of mathematics only shows itself to more patient followers." [?], meaning to gain insight there will be struggle. This is true in anything humans do, but especially true in mathematics. This is part of the beauty of doing math. Do not be discouraged.

Transcendent Beauty

Lastly, music connected us to some more greater truth: to help us with our grief, to enjoy time with our friends by singing at the top of our lungs, to calm our soul. This is called transcendent beauty. This is the beauty that moves from a specific object, idea, or musical piece to a greater truth to reveal significance in the world. [?]

In math, this is the connection of our ideas and theorems to get a better understanding of the universe. It relates us to a greater truth than ourselves. This is usually seen through the ideas surrounding the paradox between the infinite, finite, and zero. Additionally is seen through ground-breaking ideas that change our perspective on the world, such as the law of gravity, the theory of spatial relatively, the Pythagorean Theorem, and many more.

Conclusion

In this project, this transcendent beauty is the connection of the different academia disciplines that bring beauty into the world. While some are deemed more useful such as STEM, and others are deemed useless such as the humanities. There should be no superiority between different types of knowledge because they intersect more than people believe. If we focus on the attributes of one discipline than another, then we have truly lost some of understanding of the world and our humanity. Because the true beauty of knowledge lies within the complexity between reason and emotion. English is beautiful. History is beautiful. Mathematics is beautiful. These subjects are beautiful in their own ways but each has their own unique type of beauty. If we truly decided to define one form of beauty, we have lost all meaning for the word.

Mathematics is beautiful.

Appendices

.1 Definition Appendix

definitionPitch/Frequency] Frequency is the quantity the represents the number of times the object vibrates in each unit of time. To calculate this we used $f = \frac{\omega}{2\pi}$ where f is frequency, ω is the angular frequency, and where 2π is the ther period of the sine wave

definitionHarmonics/Overtones] A harmonic is a sound wave that has a frequency that is an integer multiple of a fundamental tone.

definitionAmplitude/Volume] Amplitude is the distance between the resting position and the maximum displacement of the wave. In other words, the peaks and valleys of sine waves.

definitionwav files] a file format for storing the uncompressed audio files meaning they are not smaller and they have more data.

definitionhe fundamental tone is the lowest frequency of the periodic waveform. In music this is the musical pitch of a note with the lowest overtone present.

definitionOctave] A musical interval where one note has twice the frequency of the other or half the frequency of the other.

definitionPerfect Fifth] A musical interval to a pair of notes that have a frequency ratio of 3:2 or the first note with 5 notes in between including the first note.

definitiontimbre] The character or quality of a musical sound which is distinct from its pitch and its intensity. Example: the trumpet and clarinet have two different timbres. The trumpet is bright and brassy where the clarinet is sounds more woodsy and dark.

.2 Coding Notes Appendix

```
% Finding Fourier Transform of g4

    % Loading the file

        [g4,Fs]= audioread('g4.wav');

   %Calculuating the seconds per cycle

     g4cps = 1/Fs

     % Graphing in Frequency and Amplitude in Freq domain.
     g4_L = size(g4, 1);
     g4_Fn = Fs / 2;
     g4_Fty = fft(g4/g4_L);


     g4_Fv= linspace (0 ,1, fix(g4_L/2)+1)*g4_Fn;     %Frequency Vector
     g4_Iv = 1:numel(g4_Fv);                          % Index Vector
     plot (g4_Fv, abs(g4_Fty(g4_Iv,:))*2)
        xlabel ('Frequency (hz)')
        ylabel ('Amplitude')
        title ('G4 Frequency')

% Finding Frequency coefficients Fundamental (First Harmonic)


    % Fun- Model 1
     g4_tf = excludedata(g4_Fv',abs(g4_Fty(g4_Iv,1))*2*10^(1/3), ...
         'domain',[380 400]);

     g4_funfirstfit = fit( g4_Fv', ...
        abs(g4_Fty(g4_Iv,1))*2*10^(1/3), ...
        'gauss1', ...
        'Exclude', ...
        g4_tf);

      % Fun- Model 2
     g4_tf3 = excludedata(g4_Fv',abs(g4_Fty(g4_Iv,1))*2*10^(7/24), ...
         'domain',[380 400]);

     g4_funsecondfit = fit( g4_Fv', ...
        abs(g4_Fty(g4_Iv,1))*2*10^(7/24), ...
        'gauss1', ...
```

```
        'Exclude', ...
        g4_tf3);


     % Fun- Model 3
     g4_tf3 = excludedata(g4_Fv',abs(g4_Fty(g4_Iv,1))*2*10^(1/10), ...
         'domain',[380 400]);

     g4_funthirdfit = fit( g4_Fv', ...
        abs(g4_Fty(g4_Iv,1))*2*10^(1/6), ...
        'gauss1', ...
        'Exclude', ...
        g4_tf3);

     % Fun Model 4

      g4_tf4 = excludedata(g4_Fv',abs(g4_Fty(g4_Iv,1))*2*10^(1/10), ...
         'domain',[380 400]);

     g4_funfourthfit = fit( g4_Fv', ...
        abs(g4_Fty(g4_Iv,1))*2*10^(1/5), ...
        'gauss1', ...
        'Exclude', ...
        g4_tf4);

   % Comparing Fun Models 1,2,3,4


     g4_Fv= linspace (0 ,1, fix(g4_L/2)+1)*g4_Fn;    %Frequency Vector
     g4_Iv = 1:numel(g4_Fv);                         % Index Vector
     plot (g4_Fv, abs(g4_Fty(g4_Iv,:))*2)
        xlim([380 400])
        xticks(380:5:400)
        xlabel ('Frequency (hz)')
        ylabel ('Amplitude')
        title ('G4 Frequency: First Harmonic')
      hold on
      plot (g4_funfirstfit, 'r')
      plot (g4_funsecondfit, 'b')
      plot (g4_funthirdfit, 'g')
      plot (g4_funfourthfit, 'm')

      % FUN MODEL 3 wins
        g4_funthirdfit

```

```
% Finding Frequency coefficients Second Harmonic)

        % Harm 2 Model 1

            g4_tf5 = excludedata(g4_Fv',abs(g4_Fty(g4_Iv,1))*2*10^(1/5),'domain',[775 790]);

      g4_harm2firstfit = fit( g4_Fv', ...
         abs(g4_Fty(g4_Iv,1))*2*10^(1/5), ...
         'gauss1', ...
         'Exclude', ...
         g4_tf5);

      % Harm 2 Model 2

            g4_tf6 = excludedata(g4_Fv',abs(g4_Fty(g4_Iv,1))*2*10^(7/24), ...
          'domain',[775 790]);

      g4_harm2secondfit = fit( g4_Fv', ...
         abs(g4_Fty(g4_Iv,1))*2*10^(7/24), ...
         'gauss1', ...
         'Exclude', ...
         g4_tf6);

       % Harm 2 Model 3

            g4_tf7 = excludedata(g4_Fv',abs(g4_Fty(g4_Iv,1))*2*10^(1/3), ...
          'domain',[775 790]);

      g4_harm2thirdfit = fit( g4_Fv', ...
         abs(g4_Fty(g4_Iv,1))*2*10^(1/3), ...
         'gauss1', ...
         'Exclude', ...
         g4_tf7);



      % Comparing Model 1,2,3


        g4_Fv= linspace (0 ,1, fix(g4_L/2)+1)*g4_Fn;    %Frequency Vector
      g4_Iv = 1:numel(g4_Fv);                           % Index Vector
      plot (g4_Fv, abs(g4_Fty(g4_Iv,:))*2)
         xlim([775 790])
         xticks(775:5:790)
```

```
        xlabel ('Frequency (hz)')
        ylabel ('Amplitude')
        title ('G4 Frequency: Second Harmonic')
      hold on
      plot(g4_harm2firstfit, 'r')
      plot(g4_harm2secondfit, 'b')
      plot(g4_harm2thirdfit, 'm')


      % HArm 2- MODEL 1 WINS
        g4_harm2firstfit

% Finding Frequency Coeffiicents Third Harmonic

    % Harm 3 Model 1

        g4_tf8 = excludedata(g4_Fv',abs(g4_Fty(g4_Iv,1))*2*10^(1/5), ...
            'domain',[1160 1190]);

     g4_harm3firstfit = fit( g4_Fv', ...
        abs(g4_Fty(g4_Iv,1))*2*10^(1/5), ...
        'gauss1', ...
        'Exclude', ...
        g4_tf8);

     % Harm 3 Model 2


     g4_tf9 = excludedata(g4_Fv',abs(g4_Fty(g4_Iv,1))*2*10^(1/6), ...
            'domain',[1160 1190]);

     g4_harm3secondfit = fit( g4_Fv', ...
        abs(g4_Fty(g4_Iv,1))*2*10^(1/6), ...
        'gauss1', ...
        'Exclude', ...
        g4_tf9);

     % Harm 3 Model 3

       g4_tf10 = excludedata(g4_Fv',abs(g4_Fty(g4_Iv,1))*2*10^(1/10), ...
            'domain',[1160 1190]);

     g4_harm3thirdfit = fit( g4_Fv', ...
        abs(g4_Fty(g4_Iv,1))*2*10^(1/10), ...
        'gauss1', ...
        'Exclude', ...
```

```
        g4_tf10);

    % Comparing  3Harm Models 1,2,3

        g4_Fv= linspace (0 ,1, fix(g4_L/2)+1)*g4_Fn;     %Frequency Vector
    g4_Iv = 1:numel(g4_Fv);                              % Index Vector
    plot (g4_Fv, abs(g4_Fty(g4_Iv,:))*2)
        xlim([1160 1190])
        xticks(1160:5:1190)
        xlabel ('Frequency (hz)')
        ylabel ('Amplitude')
        title ('G4 Frequency: Third Harmonic')
     hold on
     plot (g4_harm3firstfit, 'r')
     plot(g4_harm3secondfit, 'b')
     plot(g4_harm3thirdfit, 'm')

     g4_harm3firstfit


% Finding Frequency Coeffiicents 4th Harmonic

   % 4Harm Model 1

     g4_tf11 = excludedata(g4_Fv',abs(g4_Fty(g4_Iv,1))*2*10^(1/6), ...
            'domain',[1555 1575]);

    g4_harm4firstfit = fit( g4_Fv', ...
        abs(g4_Fty(g4_Iv,1))*2*10^(1/6), ...
        'gauss1', ...
        'Exclude', ...
        g4_tf11);

    % 4Harm Model 2


     g4_tf12 = excludedata(g4_Fv',abs(g4_Fty(g4_Iv,1))*2*10^(1/10), ...
            'domain',[1555 1575]);

    g4_harm4secondfit = fit( g4_Fv', ...
        abs(g4_Fty(g4_Iv,1))*2*10^(1/10), ...
        'gauss1', ...
        'Exclude', ...
        g4_tf12);

```

```
    % 4Harm Model 3


     g4_tf13 = excludedata(g4_Fv',abs(g4_Fty(g4_Iv,1))*2*10^(3/40),
        ...
           'domain',[1555 1575]);

    g4_harm4thirdfit = fit( g4_Fv', ...
       abs(g4_Fty(g4_Iv,1))*2*10^(3/40), ...
       'gauss1', ...
       'Exclude', ...
       g4_tf13);

     % 4Harm Model 4


     g4_tf14 = excludedata(g4_Fv',abs(g4_Fty(g4_Iv,1))*2*10^(1/8),
        ...
           'domain',[1555 1575]);

    g4_harm4fourthfit = fit( g4_Fv', ...
       abs(g4_Fty(g4_Iv,1))*2*10^(1/8), ...
       'gauss1', ...
       'Exclude', ...
       g4_tf14);

    % 4Harm Model 5
     g4_tf15 = excludedata(g4_Fv',abs(g4_Fty(g4_Iv,1))*2*10^(27/200)
        , ...
           'domain',[1555 1575]);

    g4_harm4fifthfit = fit( g4_Fv', ...
       abs(g4_Fty(g4_Iv,1))*2*10^(27/200), ...
       'gauss1', ...
       'Exclude', ...
       g4_tf15);

    % Comparing 4Harm Model 1,2,3,4,5

     g4_Fv= linspace (0 ,1, fix(g4_L/2)+1)*g4_Fn;     %Frequency
        Vector
    g4_Iv = 1:numel(g4_Fv);                          % Index Vector
    plot (g4_Fv, abs(g4_Fty(g4_Iv,:))*2)
       xlim([1555 1575])
       xticks(1555:5:1575)
       xlabel ('Frequency (hz)')
       ylabel ('Amplitude')
       title ('G4 Frequency: Fourth Harmonic')
```

```
      hold on
      plot(g4_harm4firstfit, 'r')
      plot(g4_harm4secondfit, 'b')
      plot(g4_harm4thirdfit, 'm')
      plot(g4_harm4fourthfit, 'g')
      plot(g4_harm4fifthfit, 'y')


      % 4Harm MODEL 5 Wins!
            g4_harm4fifthfit




% Recreating the sounds...



g4_n = 1:4800 *20;
g4_t = n/48000;



g4_f1 = 2*pi * 391.6 ;
g4_f2 = 2*pi * 782.5;
g4_f3 = 2*pi * 1176;
g4_f4 = 2*pi * 1565;

g4_sound = 0.006801*sin(g4_f1*g4_t) + 0.00221*sin(g4_f2*g4_t) +
   0.000576*sin(g4_f3*g4_t) + 0.0004165*sin(g4_f4*g4_t);



sr = 48000
g4_s=2
g4_t= linspace(0,g4_s,sr*g4_s);
```

.3 Coding Song Appendix

```
clear;

% Bb5
Bb_n = 1:4800*25;
t = Bb_n/48000;



Bb5_f1 = 2*pi * 932;
Bb5_f2 = 2*pi *  1863  ;
Bb5_f3 = 2*pi * 2795 ;
Bb5_f4 = 2*pi *  3725;

Bb5sound =   0.006052*sin(Bb5_f1*t) + 0.0007924*sin(Bb5_f2*t) + ...
0.0008232*sin(Bb5_f3*t) +  0.0001116*sin(Bb5_f4*t);


sr = 48000;
s=2.5;
t= linspace(0,s,sr*s);

Bb5_player=audioplayer(Bb5sound,sr)
playblocking(Bb5_player)

%A5

a5_n = 1:4800 * 1;
a5_t = a5_n/48000;



a5_f1 = 2*pi * 879.9 ;
a5_f2 = 2*pi * 1760;
a5_f3 = 2*pi * 2637;
a5_f4 = 2*pi * 3217;

a5_sound =  0.004759 *sin(a5_f1*a5_t) + 0.0009626*sin(a5_f2*a5_t) + ...
0.0004288 *sin(a5_f3*a5_t) + 0.0001225*sin(a5_f4*a5_t);



sr = 48000;
a5_s=0.5;
a5_t= linspace(0,a5_s,sr*a5_s);
```

```
A5_player = audioplayer(a5_sound,48000)
playblocking(A5_player)

% B5

B5_n = 1:4800 * 1;
B5_t = B5_n/48000;

B5_f1 = 2*pi *  987.7;
B5_f2 = 2*pi *  1975  ;
B5_f3 = 2*pi *  2962 ;
B5_f4 = 2*pi * 3948 ;

B5sound =   0.004095 *sin(B5_f1*B5_t) + 0.001237*sin(B5_f2*B5_t) + 0.0002443*sin(B5_f3*B5_t) +  0.0001004*sin(B5_f4*B5_t);

sr = 48000;
s=0.5;
B5_t= linspace(0,s,sr*s);

B5_player= audioplayer(B5sound, 48000)
playblocking(B5_player)

% Ab5

ab5_n = 1:4800 *25;
ab5_t = ab5_n/48000;

ab5_f1 = 2*pi * 830.6;
ab5_f2 = 2*pi * 1661;
ab5_f3 = 2*pi * 2492;
ab5_f4 = 2*pi * 3325;

ab5_sound =  0.005242 *sin(ab5_f1*ab5_t) + 0.001131 *sin(ab5_f2*ab5_t) + ...
0.0009889*sin(ab5_f3*ab5_t) + 0.0002183  *sin(ab5_f4*ab5_t);
```

```
sr = 48000;
s=2.5;
ab5_t= linspace(0,s,sr*s);


ab5_player = audioplayer(ab5_sound,48000)
playblocking(ab5_player)


% G5


g5_n = 1:4800 * 1;
g5_t = g5_n/48000;



g5_f1 = 2*pi *  784.6 ;
g5_f2 = 2*pi *  1569 ;
g5_f3 = 2*pi *  2353;
g5_f4 = 2*pi *  3137 ;

g5_sound = 0.005719*sin(g5_f1*g5_t) +  0.001078 *sin(g5_f2*g5_t) +
    ...
0.0009398*sin(g5_f3*g5_t) +  9.555e-05*sin(g5_f4*g5_t);


sr = 48000;
s=0.5;
g5_t= linspace(0,s,sr*s);

g5_player = audioplayer(g5_sound,48000)
playblocking(g5_player)

% A5


a5_n = 1:4800 * 1;
a5_t = a5_n/48000;



a5_f1 = 2*pi * 879.9 ;
a5_f2 = 2*pi * 1760;
a5_f3 = 2*pi * 2637;
a5_f4 = 2*pi * 3217;
```

```
a5_sound =  0.004759 *sin(a5_f1*a5_t) + 0.0009626*sin(a5_f2*a5_t) +
    ...
0.0004288 *sin(a5_f3*a5_t) + 0.0001225*sin(a5_f4*a5_t);



sr = 48000;
a5_s=0.5;
a5_t= linspace(0,a5_s,sr*a5_s);


A5_player = audioplayer(a5_sound,48000)
playblocking(A5_player)



% Gb5


gb5_n = 1:4800 *10;
gb5_t = gb5_n/48000;



gb5_f1 = 2*pi * 740.1  ;
gb5_f2 = 2*pi * 1480  ;
gb5_f3 = 2*pi *  2220;
gb5_f4 = 2*pi *  2962;

gb5_sound = 0.002933*sin(gb5_f1*gb5_t) + 0.002711*sin(gb5_f2*gb5_t) +
    ...
0.00159*sin(gb5_f3*gb5_t) +  0.000214*sin(gb5_f4*gb5_t);

sr = 48000;
s=1;
gb5_t= linspace(0,s,sr*s);


gb5_player = audioplayer(gb5_sound,48000)
playblocking(gb5_player)


% F5


F5_n = 1:4800 *10;
F5_t = F5_n/48000;
```

```
F5_f1 = 2*pi * 698 ;
F5_f2 = 2*pi *  1396;
F5_f3 = 2*pi *  2094 ;
F5_f4 = 2*pi *  2798;

F5_sound = 0.004752 *sin(F5_f1*F5_t) + 0.001933 *sin(F5_f2*F5_t) +
    ...
0.002127 *sin(F5_f3*F5_t) +  4.141e-05*sin(F5_f4*F5_t);


sr = 48000;
s=1;
F5_t= linspace(0,s,sr*s);

f5_player = audioplayer(F5_sound, 48000)
playblocking(f5_player)


% E5


E5_n = 1:4800 *10;
E55_t = E5_n/48000;



E5_f1 = 2*pi * 658.9 ;
E5_f2 = 2*pi *  1319;
E5_f3 = 2*pi *  1978  ;
E5_f4 = 2*pi *  2639;

E5_sound = 0.006278 *sin(E5_f1*E55_t) + 0.001277  *sin(E5_f2*E55_t) +
    ...
0.001458 *sin(E5_f3*E55_t) + 0.000107*sin(E5_f4*E55_t);


sr = 48000;
s=1;
E55_t= linspace(0,s,sr*s);


E5_player = audioplayer(E5_sound, 48000)
playblocking(E5_player)

% Db5
```

```
Db5_n = 1:4800 *10;
Db5_t = Db5_n/48000;



Db5_f1 = 2*pi *  554 ;
Db5_f2 = 2*pi *  1108 ;
Db5_f3 = 2*pi *   1660  ;
Db5_f4 = 2*pi * 2215  ;

Db5_sound = 0.007707*sin(Db5_f1*Db5_t) + 0.001013*sin(Db5_f2*Db5_t) +
    ...
0.0005376*sin(Db5_f3*Db5_t) + 0.0001168*sin(Db5_f4*Db5_t);



sr = 48000;
s=1;
Db5_t= linspace(0,s,sr*s);

Db5_player = audioplayer(Db5_sound , 48000)
playblocking(Db5_player)


%Bb5

Bb_n = 1:4800*20;
t = Bb_n/48000;



Bb5_f1 = 2*pi * 932;
Bb5_f2 = 2*pi *  1863  ;
Bb5_f3 = 2*pi * 2795 ;
Bb5_f4 = 2*pi *  3725;

Bb5sound =   0.006052*sin(Bb5_f1*t) + 0.0007924*sin(Bb5_f2*t) + ...
0.0008232*sin(Bb5_f3*t) +  0.0001116*sin(Bb5_f4*t);


sr = 48000;
s=2;
t= linspace(0,s,sr*s);

Bb5_player=audioplayer(Bb5sound,sr)
playblocking(Bb5_player)

```

```
%C6

C6_n = 1:4800 * 1 ;
c6t = C6_n/48000;



C6_f1 = 2*pi *  1046 ;
C6_f2 = 2*pi *  2095 ;
C6_f3 = 2*pi * 3137  ;
C6_f4 = 2*pi * 4182 ;

C6_sound =  0.009939 *sin(C6_f1*c6t) + 0.001487*sin(C6_f2*c6t) + ...
0.0003361*sin(C6_f3*c6t) + 0.0001857  *sin(C6_f4*c6t);


sr = 48000;
s=0.5;
c6t= linspace(0,s,sr*s);

C6_player = audioplayer(C6_sound, 48000)
playblocking(C6_player)


% B5



B5_n = 1:4800 * 1;
B5_t = B5_n/48000;



B5_f1 = 2*pi *  987.7;
B5_f2 = 2*pi *  1975  ;
B5_f3 = 2*pi *  2962 ;
B5_f4 = 2*pi * 3948 ;

B5sound =   0.004095 *sin(B5_f1*B5_t) + 0.001237*sin(B5_f2*B5_t) +
    ...
0.0002443*sin(B5_f3*B5_t) +  0.0001004*sin(B5_f4*B5_t);


sr = 48000;
s= 0.5;
B5_t= linspace(0,s,sr*s);

```

```
B5_player= audioplayer(B5sound , 48000)
playblocking(B5_player)






% Bb5


Bb_n = 1:4800*40;
t = Bb_n/48000;



Bb5_f1 = 2*pi * 932;
Bb5_f2 = 2*pi *  1863  ;
Bb5_f3 = 2*pi * 2795 ;
Bb5_f4 = 2*pi *  3725;

Bb5sound =   0.006052*sin(Bb5_f1*t) + 0.0007924*sin(Bb5_f2*t) + ...
0.0008232*sin(Bb5_f3*t) +  0.0001116*sin(Bb5_f4*t);


sr = 48000;
s=4;
t= linspace(0,s,sr*s);

Bb5_player=audioplayer(Bb5sound,sr)
playblocking(Bb5_player)





% Second Phrase:


% Bb5
Bb_n = 1:4800*25;
t = Bb_n/48000;



Bb5_f1 = 2*pi * 932;
Bb5_f2 = 2*pi *  1863  ;
```

```
Bb5_f3 = 2*pi * 2795 ;
Bb5_f4 = 2*pi *  3725;

Bb5sound =   0.006052*sin(Bb5_f1*t) + 0.0007924*sin(Bb5_f2*t) + ...
0.0008232*sin(Bb5_f3*t) +  0.0001116*sin(Bb5_f4*t);


sr = 48000;
s=2.5;
t= linspace(0,s,sr*s);

Bb5_player=audioplayer(Bb5sound,sr)
playblocking(Bb5_player)

%A5

a5_n = 1:4800 * 1;
a5_t = a5_n/48000;



a5_f1 = 2*pi * 879.9 ;
a5_f2 = 2*pi * 1760;
a5_f3 = 2*pi * 2637;
a5_f4 = 2*pi * 3217;

a5_sound =  0.004759 *sin(a5_f1*a5_t) + 0.0009626*sin(a5_f2*a5_t) +
    ...
0.0004288 *sin(a5_f3*a5_t) + 0.0001225*sin(a5_f4*a5_t);



sr = 48000;
a5_s=0.5;
a5_t= linspace(0,a5_s,sr*a5_s);


A5_player = audioplayer(a5_sound,48000)
playblocking(A5_player)

% B5



B5_n = 1:4800 * 1;
B5_t = B5_n/48000;


```

```
B5_f1 = 2*pi *  987.7;
B5_f2 = 2*pi *  1975  ;
B5_f3 = 2*pi *  2962 ;
B5_f4 = 2*pi * 3948 ;

B5sound =   0.004095 *sin(B5_f1*B5_t) + 0.001237*sin(B5_f2*B5_t) + ...
0.0002443*sin(B5_f3*B5_t) +  0.0001004*sin(B5_f4*B5_t);


sr = 48000;
s=0.5;
B5_t= linspace(0,s,sr*s);


B5_player= audioplayer(B5sound, 48000)
playblocking(B5_player)


% Ab5

ab5_n = 1:4800 *25;
ab5_t = ab5_n/48000;



ab5_f1 = 2*pi * 830.6;
ab5_f2 = 2*pi * 1661;
ab5_f3 = 2*pi * 2492;
ab5_f4 = 2*pi * 3325;

ab5_sound =  0.005242 *sin(ab5_f1*ab5_t) + 0.001131 *sin(ab5_f2*ab5_t) + ...
0.0009889*sin(ab5_f3*ab5_t) + 0.0002183  *sin(ab5_f4*ab5_t);



sr = 48000;
s=2.5;
ab5_t= linspace(0,s,sr*s);


ab5_player = audioplayer(ab5_sound,48000)
playblocking(ab5_player)


% G5
```

```
g5_n = 1:4800 *1;
g5_t = g5_n/48000;



g5_f1 = 2*pi *  784.6 ;
g5_f2 = 2*pi *  1569 ;
g5_f3 = 2*pi *  2353;
g5_f4 = 2*pi *  3137 ;

g5_sound = 0.005719*sin(g5_f1*g5_t) +  0.001078 *sin(g5_f2*g5_t) + ...
0.0009398*sin(g5_f3*g5_t) +  9.555e-05*sin(g5_f4*g5_t);


sr = 48000;
s=0.5;
g5_t= linspace(0,s,sr*s);

g5_player = audioplayer(g5_sound,48000)
playblocking(g5_player)

% A5


a5_n = 1:4800 *1;
a5_t = a5_n/48000;



a5_f1 = 2*pi * 879.9 ;
a5_f2 = 2*pi * 1760;
a5_f3 = 2*pi * 2637;
a5_f4 = 2*pi * 3217;

a5_sound =  0.004759 *sin(a5_f1*a5_t) + 0.0009626*sin(a5_f2*a5_t) + ...
0.0004288 *sin(a5_f3*a5_t) + 0.0001225*sin(a5_f4*a5_t);



sr = 48000;
a5_s=0.5;
a5_t= linspace(0,a5_s,sr*a5_s);
```

```
A5_player = audioplayer(a5_sound,48000)
playblocking(A5_player)



% Gb5


gb5_n = 1:4800 * 5;
gb5_t = gb5_n/48000;



gb5_f1 = 2*pi * 740.1  ;
gb5_f2 = 2*pi * 1480  ;
gb5_f3 = 2*pi *  2220;
gb5_f4 = 2*pi *  2962;

gb5_sound = 0.002933*sin(gb5_f1*gb5_t) + 0.002711*sin(gb5_f2*gb5_t) +
     ...
0.00159*sin(gb5_f3*gb5_t) +  0.000214*sin(gb5_f4*gb5_t);

sr = 48000;
s=1;
gb5_t= linspace(0,s,sr*s);


gb5_player = audioplayer(gb5_sound,48000)
playblocking(gb5_player)


% F5


F5_n = 1:4800 * 7.5;
F5_t = F5_n/48000;



F5_f1 = 2*pi * 698 ;
F5_f2 = 2*pi *  1396;
F5_f3 = 2*pi *  2094 ;
F5_f4 = 2*pi *  2798;

F5_sound = 0.004752 *sin(F5_f1*F5_t) + 0.001933 *sin(F5_f2*F5_t) +
   ...
0.002127 *sin(F5_f3*F5_t) +  4.141e-05*sin(F5_f4*F5_t);

```

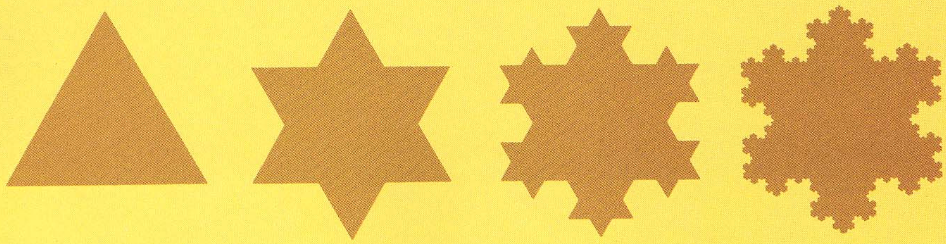
```
sr = 48000;
s=1;
F5_t= linspace(0,s,sr*s);

f5_player = audioplayer(F5_sound, 48000)
playblocking(f5_player)


% E5


E5_n = 1:4800 * 7.5;
E55_t = E5_n/48000;



E5_f1 = 2*pi * 658.9 ;
E5_f2 = 2*pi *  1319;
E5_f3 = 2*pi *  1978  ;
E5_f4 = 2*pi *  2639;

E5_sound = 0.006278 *sin(E5_f1*E55_t) + 0.001277  *sin(E5_f2*E55_t) + ...
0.001458 *sin(E5_f3*E55_t) + 0.000107*sin(E5_f4*E55_t);


sr = 48000;
s=1;
E55_t= linspace(0,s,sr*s);


E5_player = audioplayer(E5_sound, 48000)
playblocking(E5_player)

% Db5

Db5_n = 1:4800 *7.5;
Db5_t = Db5_n/48000;



Db5_f1 = 2*pi *  554 ;
Db5_f2 = 2*pi *  1108 ;
Db5_f3 = 2*pi *   1660  ;
Db5_f4 = 2*pi * 2215  ;

```

```
Db5_sound = 0.007707*sin(Db5_f1*Db5_t) + 0.001013*sin(Db5_f2*Db5_t) +
    ...
0.0005376*sin(Db5_f3*Db5_t) + 0.0001168*sin(Db5_f4*Db5_t);


sr = 48000;
s=1;
Db5_t= linspace(0,s,sr*s);

Db5_player = audioplayer(Db5_sound , 48000)
playblocking(Db5_player)




% Bb44

Bb4_n = 1:4800 * 25;
Bb4_t = Bb4_n/48000;



Bb4_f1 = 2*pi * 466.2 ;
Bb4_f2 = 2*pi *  932.2 ;
Bb4_f3 = 2*pi *  1399;
Bb4_f4 = 2*pi *   1865;

Bb4_sound = 0.009049*sin(Bb4_f1*Bb4_t) + 0.001457*sin(Bb4_f2*Bb4_t) +
    0.0005945*sin(Bb4_f3*Bb4_t) + 0.0007095*sin(Bb4_f4*Bb4_t);

sr = 48000;
s=2.5;
Bb4_t= linspace(0,s,sr*s);


Bb4_player  = audioplayer(Bb4_sound, 48000)
playblocking (Bb4_player)

% F# 4

gb4_n = 1:4800 * 1;
gb4_t = gb4_n/48000;



gb4_f1 = 2*pi * 369.7 ;
gb4_f2 = 2*pi * 739.5  ;
gb4_f3 = 2*pi * 1109;
```

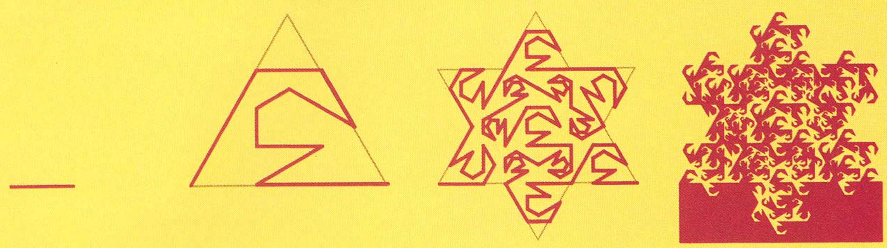
```
gb4_f4 = 2*pi * 1480 ;

gb4_sound = 0.005153 *sin(gb4_f1*gb4_t) +  0.003346*sin(gb4_f2*gb4_t)
    + ...
0.001624*sin(gb4_f3*gb4_t) + 0.0001257*sin(gb4_f4*gb4_t);

sr = 48000;
s=0.5;
gb4_t= linspace(0,s,sr*s);

gb4_player = audioplayer(gb4_sound,48000)
playblocking(gb4_player)

% G 4

g4_n = 1:4800 * 1;
g4_t = g4_n/48000;



g4_f1 = 2*pi * 391.6 ;
g4_f2 = 2*pi * 782.5;
g4_f3 = 2*pi * 1176;
g4_f4 = 2*pi * 1565;

g4_sound = 0.006801*sin(g4_f1*g4_t) + 0.00221*sin(g4_f2*g4_t) + ...
0.000576*sin(g4_f3*g4_t) + 0.0004165*sin(g4_f4*g4_t);



sr = 48000
g4_s= 0.5
g4_t= linspace(0,g4_s,sr*g4_s);

g4_player = audioplayer(g4_sound, 48000)
playblocking(g4_player)


%B4


n = 1:4800 *10;
t = n/48000;



b4_f1 = 2*pi * 493.8 ;
b4_f2 = 2*pi *  988.5  ;
```

```
b4_f3 = 2*pi * 1481 ;
b4_f4 = 2*pi *  1973;

b4_sound =   0.0011*sin(b4_f1*t) + 0.0003437*sin(b4_f2*t) + ...
0.0006619*sin(b4_f3*t) + 0.0005551*sin(b4_f4*t);



sr = 48000;
s=1;
t= linspace(0,s,sr*s);

b4_player  = audioplayer(b4_sound, 48000)
playblocking (b4_player)


line1 =[Bb5sound,a5_sound,B5sound,ab5_sound,g5_sound, a5_sound,
   gb5_sound, ...
    F5_sound, E5_sound, Db5_sound, Bb5sound, C6_sound,B5sound,
       Bb5sound];

line2 = [Bb5sound,a5_sound,B5sound,ab5_sound,g5_sound, a5_sound,
   gb5_sound, ...
    F5_sound, E5_sound, Db5_sound, Bb4_sound, gb4_sound, g4_sound,
       b4_sound];

song=[line1, line2];
```

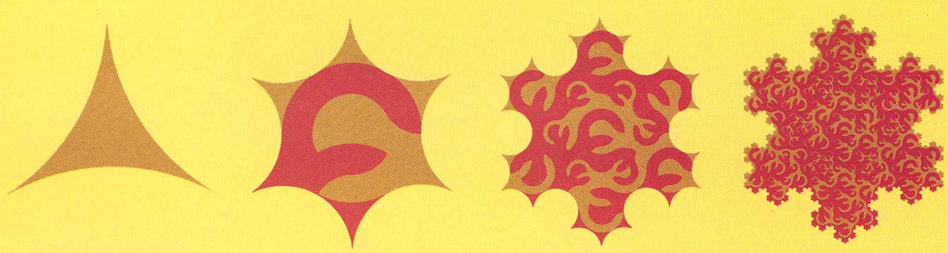
.4 Fractals: Koch Snowflake and Snowflake Sweep



3a: four stages in the construction of the Koch snowflake



3b: four stages in the construction of a snowflake sweep



3c: rounded approximations of 3a and 3b, superposed

.5 Honors Symposium Speech

[12pt]article
enumitem [margin=1in]geometry
setspace natbib hyperref xcolor

mathabx, amssymb, relsize, tikz, mathtools, amsmath, amsthm, enumitem, hyperref, graphicx,

float, bbm, fancyhdr [utf8]inputenc [english]babel [mathscr]euscript setspace

Theorem[section] Corollary[theorem] Lemma[theorem] Problem

Im Imcurl$curl$

Framing Reality through the Lens of Creative Mediums

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Context

Through this we are examining the what it means to be human by examining how humans are able to experience the world through creative mediums. As a panel, we explored the effects of immersive art and how sensory beauty impacts the creation of human experiences. Furthermore, panelists analyze the framework of beauty to better understand how humans perceive meaning.

Introduction

Thank you for coming to our panel.

I am here both as a musician and as a mathematician.

As a musician, I am trained to use music to entertain My role is to connect the audience to different emotions and various stories. Ultimately, my role is to bring beauty into the world.

As a mathematician, I am trained to use logic and reasoning to solve problems. My role is to connect complex ideas and singular solutions. Ultimately, my role is to provide insight.

Why do we view music as beautiful, but not mathematics?

Behind me, you see Claude Debussy's, famous flute piece, *Syrinx*. In the flute world, this is renowned as a beautiful piece of musical literature. It is known for its narrative storytelling of the Greek legend regarding Syrinx and Pan and for its mesmerizing melodies. .

For this project, I recreated the first two measures of this piece by decomposing sound into mathematical equations. Let me play this for you.

Before, we dive deeper in, let me play for you a musical interpretation.

While, I am playing my musical interpretation for you, observe the code behind me. Realize that this is **a** mathematical representation of what I am playing.

Math Context Vocabulary

I have just played my musical interpretation of this mathematical composition.

Now, Let's define some math vocab.

I will be talking about these three main ideas: Frequency, Harmonics, and Amplitude:

a. First is Frequency. Frequency is the number of times a sound wave repeats over a period of time.

b. Second is Harmonics. Let's consider one note. As it plays, think of one knot being ties into one piece of rope. This is called our first harmonic. We can tie a smaller second knot. This is our second harmonic. Theoretically we can tie an infinite amount of knots, but who wants to? I don't. On flute, there are 9 playable harmonics, or knots,. In this project, I limited our harmonics to 4 because there were 15 notes. In terms our our knot analogy, I have 15 disconnected pieces of rope with 4 notes in each piece.

c. Lastly is Amplitude. Amplitude is how loud or soft the music is. We measure amplitude from the peaks and valleys of sound waves.

Now that we have defined frequency, harmonics and amplitude, let's explore this means.

Using a branch of mathematics called Fourier Analysis, we are able to use a technique called Fourier Transform to graph each note via their frequencies and amplitudes.

Math Modelling Go!

We learned the key math concepts, so let's run through the process in which I created my mathematical composition.

In the scope of this project, I looked at the first two measures of *Syrinx*, and counted 15 different notes. Next, I recorded myself playing each note in Pro Tools, an audio software tool. I played each note for about 4 seconds, and ended up with 15 sound files. Each sound file represents one note of the piece.

After I gathered my sound files, I imported them in a computer program called MATlab. Matlab is designed for sound analysis which sounds perfect for us.

After importing the sound files into Matlab, I used Fourier Transform, to find Fourier coefficients. These Fourier coefficients are used to specify the amplitudes of our sound waves.

Recall that, one note has 4 harmonics. So for one note, we are finding the 4 separate coefficients for each of the harmonics. We now repeat this process for the other notes. Since we had 15 notes with each note having 4 harmonics, we found 60 unique Fourier Coefficients.

What we have now are our frequencies and amplitudes but no record of the duration of each note. So I added time to my frequency and amplitude graph to change the duration of my notes. I either increased the time interval if I wanted the note to be longer, or decreased it if I wanted the note to ve shorter. Here, I have now created a sense of rhythm.

Time was my missing component in creating the equation, but since we have it, able to begin the process of recreating the sound. First we created an equation of each note, composed of the sum of the 4 harmonics with each harmonic being multiplied by its unique Fourier Coefficient. Then using Matlab, I coded a way for the program to take my equation and play the sound. Resulting in the sound for each note. Afterwards, I take the code for each note and order it, based off how it appears in the piece. Then I adjusted the time interval to create a sense of rhythm. We have now just created my mathematical composition of *Syrinx*.

Glimpse Mathematical Beauty

Now, back to original question: why is music considered beautiful but mathematics is not?

Especially when music can be written in terms of mathematics.

To get us thinking more about mathematical beauty, I quote G.H Hardy,

"It may be very hard to define mathematical beauty, but that is just as true of beauty of any kind- we may not quite know what we mean by a beautiful poem but does that not prevent us from recognizing one when we read it?"

With this in mind, I now present the Webster definition of beauty.

Beauty is the quality or aggregate of qualities in a person or thing that gives pleasure to the senses or pleasurably exalts the mind or spirit.

Instinctively, we think of music. We think of how music makes us but we hardly think about how math makes us feel. If we do, it is usually in a negative context. Thus, let us reframe beauty in the context of mathematics

"Exalting the mind and spirit" applies directly to mathematics. We exalt the mind, with complex ideas, and challenging problems. We strive to connect ideas and solutions. We as mathematicians crave meaning out of the theorems we prove.

A famous example includes Euclid's Proof of the Infinitude of Primes, which states that given any collection of prime numbers there is at least one prime number that is not part of this collection. In mathematics, this is beautiful for two reasons: for the idea it conveys and how elegant and simple the proof is.

A huge appeal of mathematical beauty is the idea of being able to reason and bring insight about complex ideas.

But you ask, "What if someone's taste is different than what society deems as beautiful? "

Objectivism and Subjectivity

This question leads us to two different ways to aesthetically define beauty. These two ways are Aesthetic Subjectivity and Aesthetic Objectivity.

a. Aesthetic Subjectivism is the belief that aesthetic judgements such as beauty are not facts but rely on the observer. This is essentially the argument that beauty is in the eye of the beholder

   If this were truly the case, this would mean that a toddler's finger painting would be more beautiful than the Mona Lisa. I don't mean to offend any parents in the room, but we can all agree that regardless of parental sentimental value the Mona Lisa is objectively a better painting, majority considered to be a masterpiece of the art world.

This bring us to the another way of aesthetically defining beauty:

b. Aesthetic Objectivity. This is the belief that aesthetic qualities are properties of objects that are independent of an observer's awareness.

Both subjectivity and objectivity play a vital role in the human's perspective of beauty.

When sharing this topic of mathematical beauty to others, I constantly hear people belittle math.

"It's too hard"

Good for you, I can never do that.

Bless your soul

To this, I respond with quote from Imannuel Kant's Critique of Judgement. "the judgement of taste is subjective." Meaning that mathematical beauty still exists regardless whether society deems it as beautiful or not.

Frameworks of Beauty

Then, let us discuss the why mathematics is beautiful.

Francis Su discusses the 4 main types of beauty: sensory, wondorous, insightful, and transcendent.

a. First is **Sensory beauty**. This is the beauty that deals with the senses: sight, touch, taste, smell, and sound.

In math, sensory beauty present itself in nature and art.

In nature, mathematics is present through patterns. The famous example is the Fibonacci sequence. This sequence originates from how rabbits reproduced given that each pair has one male and one female. This one question resulted in seeing this pattern everywhere in nature. It is seen on the seeds of sunflowers, cauliflower, and pine-cones. Futhermore, it is seen storms and how they spiral and move. Even the human body follows this pattern. With our 1 nose, 2 eyes, 3 segemts to each limb and 5 fingers. Mathematics is seeing and applying patterns in different contexts.

In art, mathematics is presented in the creation of parallel lines in artwork. In the Last Supper, the parallel lines on the ceiling are constructed by created an intersection point at infinity. This gives us the illusion of parallel when they don't look like it.

Specifically in this project, we appeal to sensory beauty of math through sight and sound. We can see the physical representation of our equations, and we can also hear them.

b. Next, is **Wondorous Beauty**. Wondorous Beauty is being awed and curious about something.

Generally in math, we are constantly curious about the world. We pose questions by limiting our assumptions and to see what happens next.

In this project, we can subscribe this to sparking one's curiosity. Throughout this project, there were many quesiions with one common theme: a way to solve new problems. This was a constant throughout the entire process of creating a computer generated sound modelling music. Additionally, the idea allows us as humans to continue to become lifelong learners.

Sensory Beauty relates to Wondrous Beauty because it can spark our curiosity, but both forms of beauty are independent types of beauty. Both forms of beauty can lead to **insightful beauty**.

c. **insightful beauty** is the beauty of how we understand the world based off the insights we gather. This type of beauty is where the stereotype of mathematics as being cold and devoid of emotion originates because this beauty focuses on the art of understanding through reason.

In math, insightful beauty is constantly applied.

To figure out seating for 16 people using trapezoidal desks, totally not applicable to me. To figuring out the least amount of pizza to order to feed enough people. To understanding the mathematics behind physics, engineering. To know how to to safely build bridges, roads, desks, chairs. To analyze trends in data to gleam insights. Note that, insights can be instantaneous but also insights can also happen over time.

Furthermore insightful beauty in mathematics is best exemplified through the art of proof. As a mathematician, we try to create the simplest and most insightful proofs to truly understanding the mechanics of the world.

This is the power to communicate ideas effectively.

During this project, there were obstacles to overcome:

a. Learning a new coding language

b. creating code to find coefficients

c. defining on what the best model is

d. how to create rhythm using code

e. Understanding a new higher level math such as Fourier Analysis

To say the least, I struggled. These were all new skills I wanted and needed to learn. We learn these skills only through preserving through the struggle. We continued because it was fun to think about different results and implications.

d. Ultimately these results lead to some sort of greater truth. This is **transcedent beauty**. This is the beauty that moves from a specific object, idea, or result to a greater truth about the world.

In math, this is the connection of our ideas and theorems to get a better understanding of the universe. It relates us to a truth greater than ourselves. Usually seen through the abstract ideas regarding infinity and zero, but also in ground-breaking research. This ground-breaking research are ideas that change our perspective on the world. Examples include law of gravity, the theory of spatial relativity, the Pythagorean Theorem, Euclid's Infinitude of Primes, and many more.

Conclusion

Specifically mathematical transcendental beauty is recognizing that math is a different perspective of observing and learning about the world. To be truly human, a person must have the ability to understand and percieve beauty in different contexts.

In modern society, There is an emphasis on thinking logically which results in an emphasis on STEM fields. But reflecting, critical thinking and empathy are all vital aspects of being human. Without English or History, we are unable to to learn from mistake from history or to relate to characters. In more applicable terms: to learn our own past mistakes, and to relate to other people.

Sterotyping different aspects of knowledge reduces our ability to be human. Because our humanity is in the intersection of knowledge surrounding STEM and Humanities.

This project exemplifies the commonalities between math and music to relate the beauty of math to people who do not find math beautiful. To find math beautiful is the Resistance against one common definition of beauty.

I strongly urge you all, to no discredit other perspectives of beauty. To truly humble ourselves to learning more. As a society, if we solely decide on one form of beauty, we have done ourselves an injustice. Because we have lost a significant understanding of our universe.

English is Beautiful.

Horror is Beautiful

Video Games are Beautiful.

Mathematics is Beautiful.

Thank you.